Freeze Column Examples

1 Introduction

This example discusses two GeoStudio TEMP/W example files related to the Neumann thaw and freeze analyses in one dimension, with and without infinite elements. The intent of the example is to verify the phase change algorithm in TEMP/W against a known solution. The two example files are named:

- Freeze Column with no Infinite Elements
- Freeze Column with Infinite Elements

2 Feature highlights

GeoStudio feature highlights include:

- Transient heat flow
- Phase change in TEMP/W
- Verification of formulation

3 Discussion

Graphical and closed-form solutions for these types of problems are available when a step temperature is applied at the surface. The thaw and freeze depths can be computed using a Neumann solution (Nixon and McRoberts, 1973), when the following parameters are defined:

Ku = thermal conductivity of unfrozen soil,

Kf = thermal conductivity of frozen soil,

Cu = volumetric heat capacity of unfrozen soil,

Cf = volumetric heat capacity of frozen soil,

L = volumetric latent heat,

Tg = uniform initial ground temperature, and

Ts = applied constant surface temperature.

Problem definition

Figure 3-1 illustrates the problem and presents the parameters selected for verifying TEMP/W.

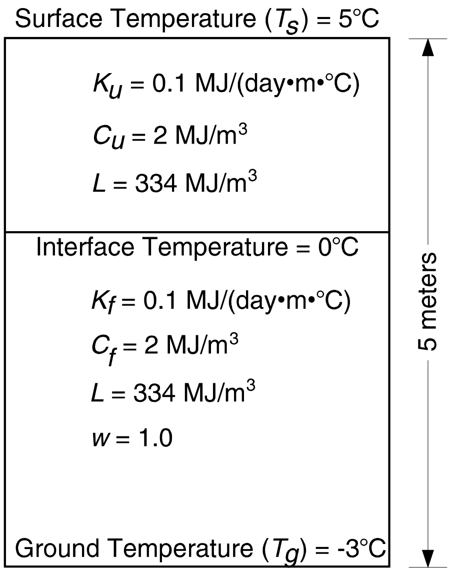


Figure 3-1 One dimensional thaw and freeze problem

In this example, the thermal conductivity, (K), and volumetric heat capacity, (C), are the same for the frozen and unfrozen zones, and the volumetric water content is 1.0. The latent heat, (L), is therefore equal to the latent heat of water. The thermal units in the original example are MJ, but this is not an explicit option in TEMP/W. Therefore, in TEMP/W the units shown will substitute H (for heat) wherever the heat units are required.

The closed-form solution assumes that all the latent heat is absorbed or liberated at a temperature of 0°C. To simulate such a process, the simplified thermal model is used in TEMP/W. No unfrozen water content is required in this case.

Solution

The closed-form solution for the advance of the thawing and freezing front for this problem was derived by Neumann around 1860, and is given by Carslaw and Jaeger (1947) as:

$$X = \alpha \sqrt{t}$$

where:

X	=	depth of the thawing or freezing fronts,
t	=	elapsed time, and
α	=	a constant which is a function of material properties and boundary conditions.

The relationship of α to the various parameters is:

$$\frac{\alpha}{2(k_u)^{1/2}} = f\left\{ste,\ \left[-\frac{T_g K_f}{T_s K_u}\left(\frac{k_u}{k_f}\right)^{1/2}\right]\right\}$$

where Ste, the Stefan number, is defined as the ratio of sensible heat to latent heat by:

$$Ste = \frac{C_u\, T_s}{L}$$

The parameters ku and kf are the diffusivities of unfrozen and frozen soil, and are defined as the thermal conductivity divided by the volumetric heat capacity. In equation form:

$$k_u = \frac{K_u}{C_u}$$

$$k_f = \frac{K_f}{C_f}$$

Nixon and McRoberts have presented a graphical solution to the Neumann equation, as shown in Figure 3-2.

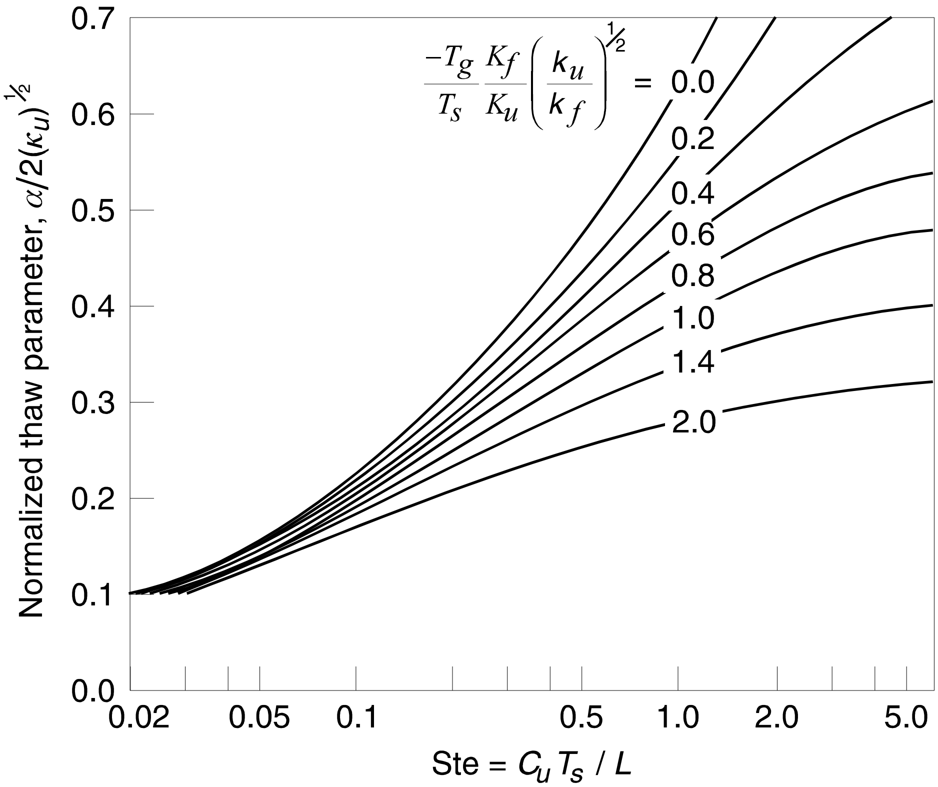


Figure 3-2 Solution of the Neumann Equation (after Nixon and McRoberts)

Depth of thawing front

For this example, since ku equals kf and Ku equals Kf, the parameter $-\frac{T_g}{T_s}\frac{K_f}{K_u}\left(\frac{k_u}{k_f}\right)^{1/2}$ is reduced to a ratio of $-\frac{T_g}{T_s}$. In the case of thawing, Ts = 5°C and Tg = -3°C. Therefore, the ratio is:

$$-\frac{T_g}{T_s} = \frac{-\ \ -3}{5} = 0.6$$

The variable Ste is calculated as:

$$Ste = \frac{C_u\ T_s}{L} = \frac{2.0\quad 5}{334} = 0.03$$

Using these two known variables, the normalized thaw parameter $\frac{\alpha}{2\sqrt{k_u}}$ can be obtained from the graph in Figure 3-2 as 0.1. We can then calculate α as:

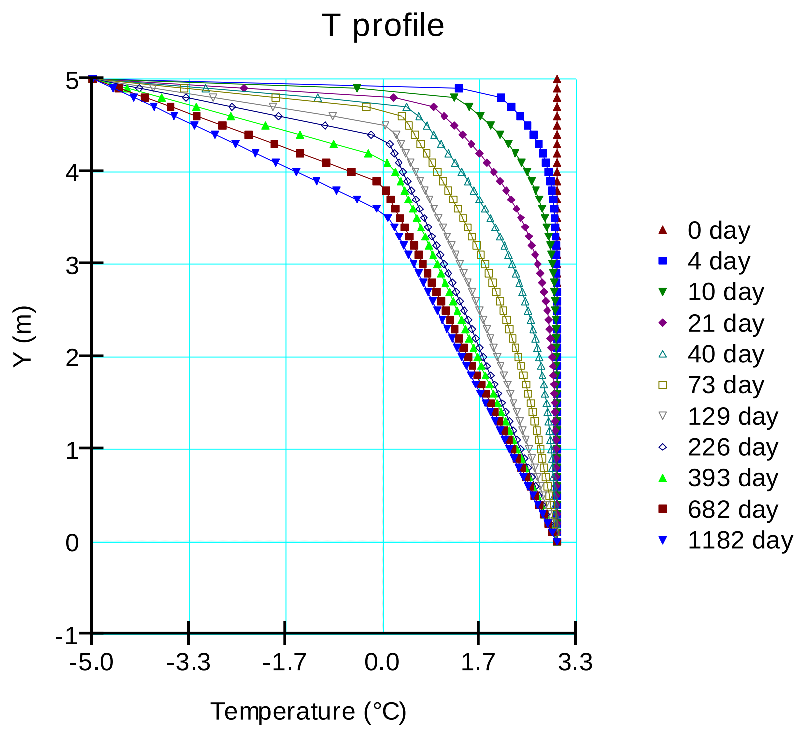
$$\alpha = 2\sqrt{k_u}\quad 0.1\ \ = 2\sqrt{0.05}\quad 0.1\ \ = 0.045$$

With α known, the depth of the thawing front can be computed.

This problem is simulated in TEMP/W with 35 elements and 178 nodes. A TEMP/W solution to the problem is also obtained by using an infinite element at the bottom boundary. The initial temperature of the entire column is defined at -3°C using the Draw Initial Conditions command in DEFINE. The ground surface is specified as a constant temperature boundary, T = 5°C.

Figure 3-3 compares the analytical solution, (Neumann solution), with the TEMP/W computed results. The solid line is the analytical solution and the data points represent the thaw depth computed by TEMP/W. As indicated in the comparison, excellent agreement exists between the graphical Neumann solution and the TEMP/W results.

The TEMP/W solution using the infinite element appears to compare even closer to the analytical solution than the solution without the infinite element, especially after a large elapsed time. This is consistent with the fact that the analytical solution is developed for a semi-infinite column, which can be better simulated with an infinite element at the bottom boundary.



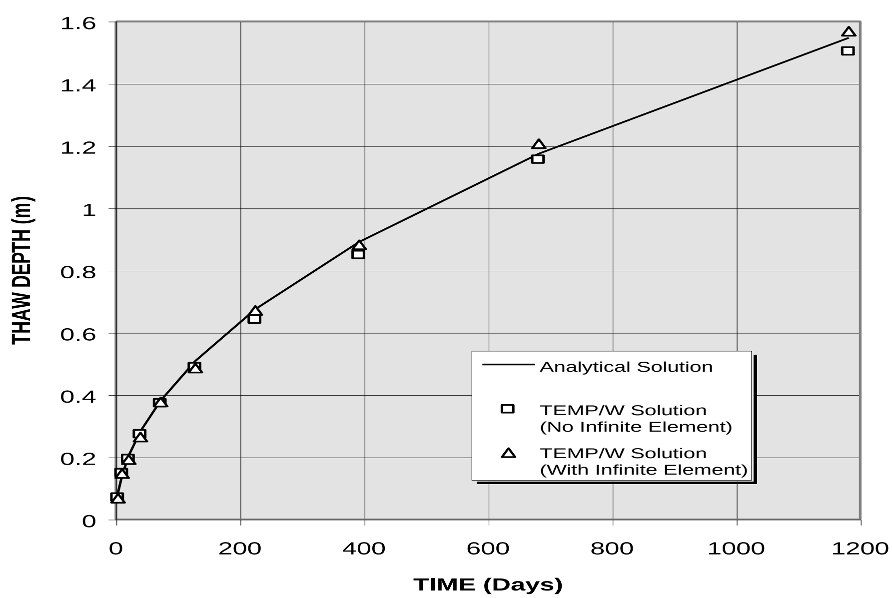


Figure 3-3 Comparison of the Neumann and TEMP/W results for thaw depth vs time

Depth of freezing front

The one-dimensional thaw problem in the previous section is repeated here with the initial ground temperature at +3°C and the initial surface temperature at -5°C. In all other respects, this problem is identical to the thaw problem in the previous section.

In the case of freezing, Ts = -5°C and Tg = 3°C. Therefore, the -Tg/Ts ratio is:

$$-\frac{T_g}{T_s} = \frac{-\ 3}{-5} = 0.6$$

The variable Ste is calculated as:

$$Ste = \frac{C_u\ T_s}{L} = \frac{2.0 \quad -5}{-334} = 0.03$$

Using these two known variables, the normalized thaw parameter $\frac{\alpha}{2\sqrt{k_u}}$ can be obtained from the graph in Figure 3-2 as 0.1. We can then calculate α as:

$$\alpha = 2\sqrt{k_u} \quad 0.1 \ = 2\sqrt{0.05} \quad 0.1 \ = 0.045$$

With α known, the depth of the thawing front can be computed.

The TEMP/W solution to the problem is also obtained using an infinite element along the bottom boundary. The initial temperature of the entire column is defined at 3°C using the Draw Initial Conditions command in DEFINE. The ground surface is specified with a constant temperature boundary, T = -5°C.

The solutions for freezing and thawing must be identical when the material properties for both frozen and unfrozen soils are the same. Figure 3-4 shows the frozen depth versus time functions for the Neumann and TEMP/W solutions. A comparison of Figure 3-3 and Figure 3-4 reveals that the computed solutions are identical for both the thawing and freezing cases. This verifies that TEMP/W is working properly for both thawing and freezing simulations.

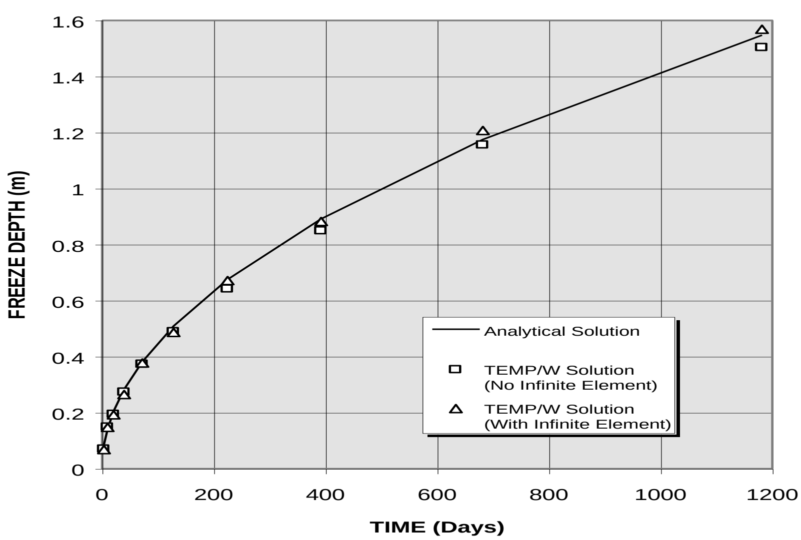


Figure 3-4 Comparison of Neumann and TEMP/W results for freeze depth vs time