# Salt Flow Example

## 1 Introduction

Total hydraulic head consists of elevation head and pressure head. Pressure head is dependent on the density of the pore fluid, which is a function of both temperature and chemical composition. For density-dependent groundwater flow problems, the driving forces for flow include the hydraulic gradient, as well as a buoyancy force term caused by variations in density. Consequently, spatial variations in salinity generate a total hydraulic head gradient that governs density-dependent groundwater flow.

The objective of this detailed example is to verify, by means of some simple calculations, the hydraulic head and groundwater fluxes calculated by CTRAN/W for coupled one-dimensional density-dependent groundwater flow problems.

## 2 Background

Only a brief summary of the theory for density-dependent flow is provided here. A more comprehensive review can be found in the CTRAN/W Engineering book and the literature (e.g. Fetter, 1998; Frind, 1982a, b). As noted in the introduction, the pore fluid density affects the pressure head component of total hydraulic head. Consider the two point measurements of hydraulic head shown in Figure 1. The actual hydraulic pressure at $P_1$ in the saline-water aquifer is

$$P_1 = \rho_s g h_s \quad [1]$$

where

$\rho_s$ = saline pore fluid density;

$g$ = acceleration due to gravity; and,

$h_s$ = height of saline water in the piezometer.

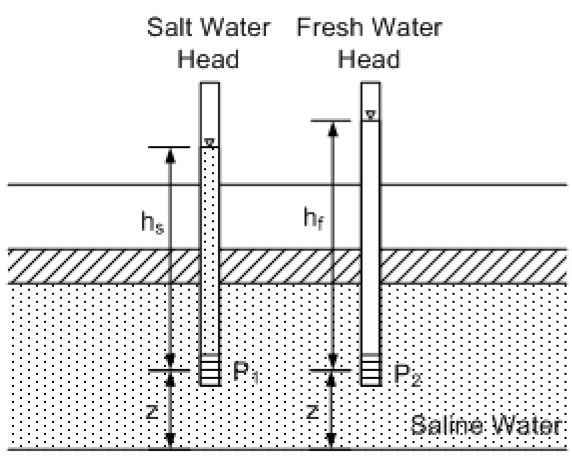


**Figure 1 Equivalent fresh water head (after Fetter, 1988)**

Assume that another piezometer was completed in the saline-water aquifer at $P_2$, but that it was filled with fresh water. The hydraulic pressure at $P_2$ is

$$P_2 = \rho_f g h_f \qquad [2]$$

where

$\rho_f$ = freshwater density; and,

$h_f$ = height of fresh water in the piezometer.

The piezometers are completed at the same elevation, so $P_1$ must equal $P_2$. Setting Eq. [1] equal to Eq. [2] results in the following relationship between the fresh water pressure head and saline-water pressure head:

$$h_f = \frac{\rho_s}{\rho_f} h_s \qquad [3]$$

The equivalent fresh water total head can now be calculated as the fresh water pressure head plus the elevation head $z$:

$$\psi = \frac{\rho_s h_s}{\rho_f} + z \qquad [4]$$

Use of the equivalent freshwater total head for density-dependent groundwater flow problems requires the general Darcy equation to be recast as

$$q_i = -K_{ij}\left(\frac{\partial \psi}{\partial x_j} + \rho_r n_j\right) \qquad [5]$$

where

$K_{ij}$ = hydraulic conductivity tensor;

$n_j = 1$ indicates the vertical direction and $n_j = 0$ indicates the horizontal direction; and,

$\rho_r$ = relative density defined as:

$$\rho_r = \frac{\rho_s}{\rho_f} - 1 \qquad [6]$$

The indices $i$ and $j$ are varied as $i = j = 0$, which corresponds to the horizontal flow component (note: the horizontal flow component is unaffected by the second term because $n_j = 0$ ), and $i = j = 1$ which corresponds to the vertical flow direction.

In this form, Darcy's equation contains two dynamic driving forces: the hydraulic gradient calculated from the equivalent freshwater total head and the buoyancy force ( $\rho_r$ term). The buoyancy force is often referred to as the body force and represents the additional gradient caused by variations in density.

The implementation of Eq. [5] requires knowledge of the pore fluid density $\rho_s$ , or more specifically the ratio of $\rho_s / \rho_f$ . The pore fluid density is a function of the concentration of the dissolved solute. For isothermal conditions and a range of concentrations up to that of seawater, there is essentially a linear relationship between fluid density and concentration (Figure 2) that can be written as

$$\rho_s = \rho_f (1 + \gamma c) \quad [7]$$

where $c$ is the concentration normalized to the maximum concentration ($C_{max}$) and ranging from 0 to 1.0, and $\gamma$ is the contaminant density contrast equal to

$$\gamma = \frac{\rho_{\max}}{\rho_f} - 1 \quad [8]$$

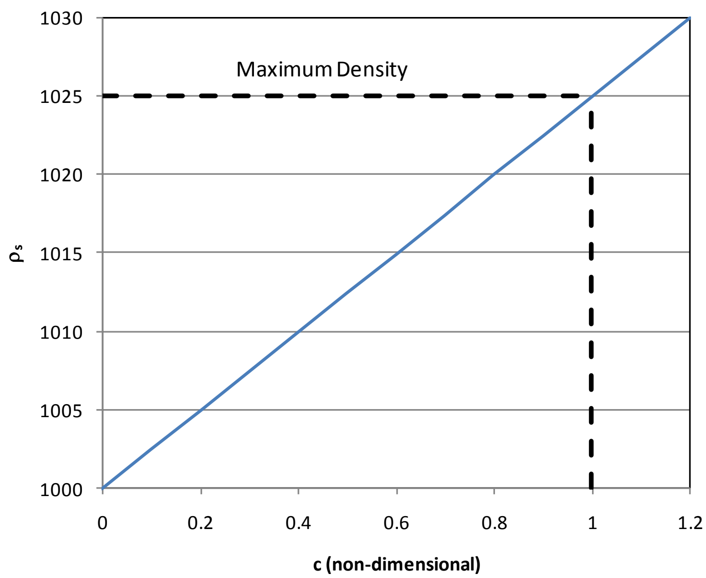


**Figure 2  Relationship between density and concentration**

Substituting Eq. [7] into [6] yields the following for relative density ( $\rho_r$ ):

$$\rho_r = \gamma c \quad [9]$$

By substituting Equation [9] into  [5], Darcy's Law can be written as

$$q_i = -K_{ij} \left( \frac{\partial \psi}{\partial x_j} + \gamma c n_j \right) \quad [10]$$

In this form, the contaminant density contrast ( $\gamma$ ) and normalized concentration ($c$) are required to compute seepage fluxes.  In SEEP/W, the user is required to specify a relative density at a reference concentration under the KeyIn Analysis dialogue box.  The relative density (not to be confused with Eq. [6]) is simply the ratio of $\rho_{\max} / \rho_f$ in Eq. [8] at a maximum concentration (i.e. the reference concentration).  Accordingly, CTRAN can compute the normalized concentration ($c$) using the reference concentration ($C/C_{ref}$).    For example, assume that the user input relative density was 1.025 (i.e. seawater) at a reference concentration of 10,000 g/m$^3$.  If the concentration at a point in the model domain was 5000 g/m$^3$, the relative concentration is $c = 0.5$ and $\gamma$ = 0.025.

For density-dependent problems, the constitutive relationship (i.e. Eq. [5]), and therefore the groundwater velocity, is a function of concentration via the fluid density ( $\rho_s / \rho_f$ ). Similarly, the governing equation for solute transport (refer to the CTRAN Engineering book) is a function of the groundwater velocity through the advective transport term. The equations are therefore coupled through density and velocity.

In the CTRAN/W and SEEP/W formulation, SEEP/W computes the velocity and passes this information to CTRAN/W. CTRAN/W then computes the concentration and then passes the concentration values back to SEEP/W. The two programs pass this information back and forth until there is no further change in H and C, or in other words, until the solution has converged.

## 3 Boundary Conditions and Material Properties

The Salt Flow gsz file includes a density-dependent SEEP/W analysis and a density-dependent CTRAN/W analysis. Adding a density-dependent SEEP/W analysis automatically generates the associated CTRAN/W analysis (or vice versa). The time step information and convergence criteria are specified in the CTRAN/W KeyIn Analysis. A screen capture of the KeyIn Analyses dialogue box is presented in Figure 3. The relative density was specified as 1.025 (i.e. seawater) at a reference concentration $c = 1.0$ g/m$^3$.

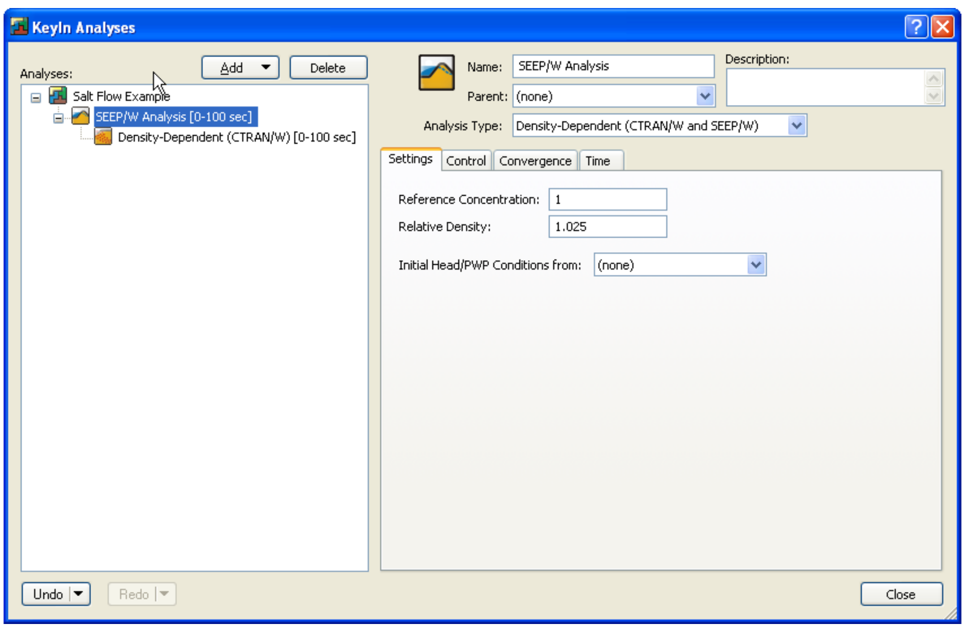


**Figure 3 Model structure for the Salt Flow Example**

The example file includes two model domains: a vertical column and a horizontal column. Each column is 1 m in length and 0.1 m in width with a mesh that consists of 20 elements and 22 nodes. The hydraulic conductivity and porosity of the soil is 1 m/sec and 0.35, respectively. For the vertical column, the top boundary is set as a constant head of 1 m (H = 1 m). The left and right boundaries of the horizontal column are set to total hydraulic heads of 0.2 m and 0.1 m, respectively.

The soil is assigned a coefficient of diffusion (D) of $1\times10^{-5}$ m$^2$/sec. The longitudinal and transverse dispersivity values were arbitrarily set to a low value ($1\times10^{-20}$ m$^2$/sec). A unit concentration of 1 g/m$^3$ is applied to both columns. Accordingly, the concentration throughout the model domain is equal to the user-defined reference concentration (i.e. c = 1.0) and the body force term will equal 0.025. The initial pore water conditions are specified using a piezometric line, while the initial concentrations are defined using the 'activation concentration' under KeyIn Materials. The analyses were run for an elapsed time of 100 seconds with one time increment.

# 4 Results and Discussion

## 4.1 Vertical Column

Figure 4 presents a profile of the equivalent fresh water head computed by SEEP/W for the vertical column. The fresh water head at the bottom of the column is 1.025 m, which is equal to the hydraulic pressure (10.052 kPa) divided by the unit weight of fresh water (9.81 kPa/m). Although there is an upwards hydraulic gradient of $\partial\psi/\partial x_1 = -0.025$ in the column, the vertical groundwater velocity is zero. This occurs because the upward hydraulic gradient is counterbalanced by the downward body force $\gamma c = 0.025$ in Eq. [10].

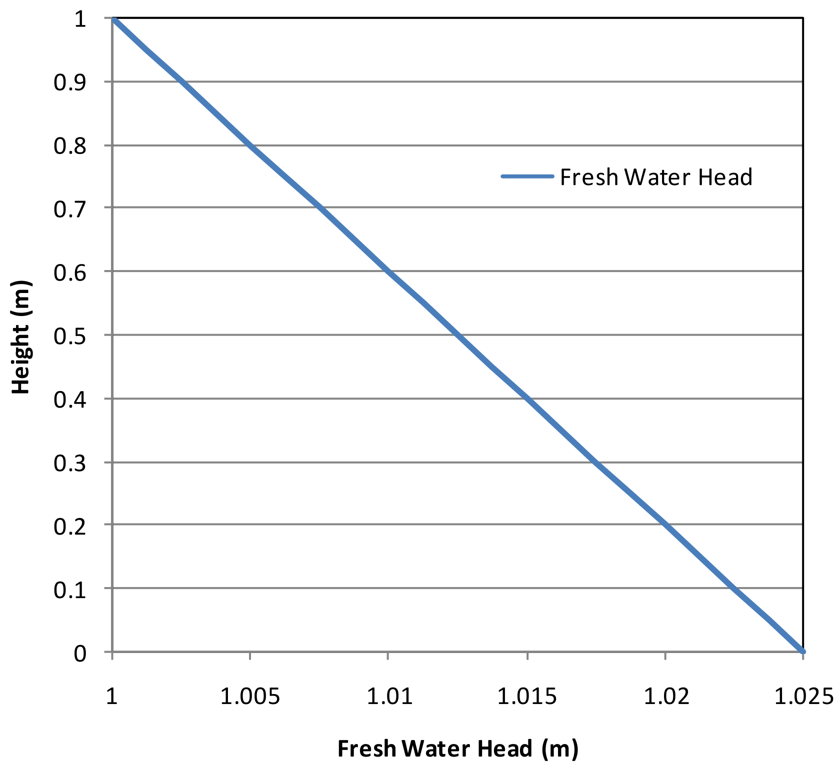


**Figure 4 Vertical Profile of Fresh Water Head**

## 4.2 Horizontal Column

Figure 5 presents the contours of equivalent fresh water head along with values at the four corners of the horizontal column. The fresh water hydraulic heads computed at points A, B, C, and D are consistent with Eq. [3]. For example, the salt water head along the left boundary was set to 0.2 m, so the pore-water pressure at point A is equal to 1.005525 kPa (i.e. $\rho_s g h_s$ = (1025 kg/m$^3$)(9.81 m/s$^2$)(0.2 – 0.1)m). The equivalent freshwater head is calculated as 0.2025 m using Eq. [3]. There is an upward fresh water hydraulic gradient of 0.025 m/m across the entire column, as was the case with the vertical column.

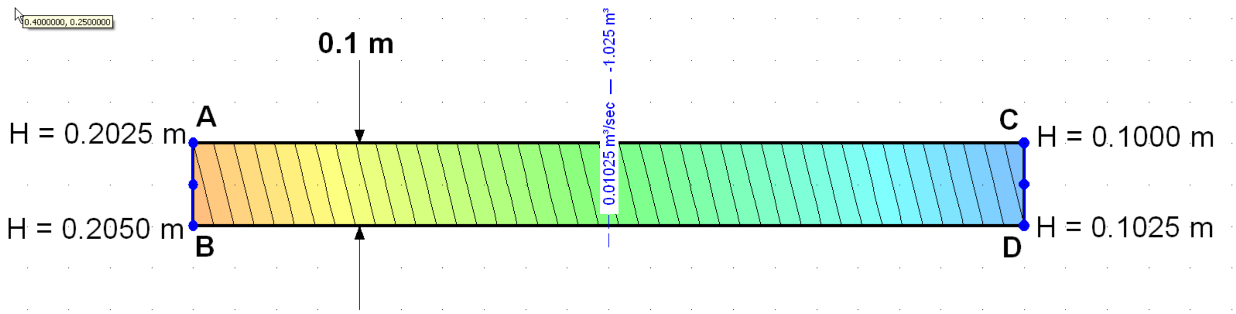


**Figure 5 Contours of Equivalent Fresh Water Total Head**

Results for a Gauss region in the model domain are shown in Figure 6. The upward fresh water hydraulic gradient is counterbalanced by the downward body force, so the vertical gradient and the liquid y-velocity throughout the model domain are zero. The fresh water hydraulic gradient in the horizontal direction across the column at all elevations is 0.1025. Given a hydraulic conductivity of 1.0 m/s and a column length of 1.0 m, the calculated groundwater flux is 0.1025 m/s. The cumulative flux over an elapsed time of 100 seconds is 1.025 $m^3$, as shown on the flux section in Figure 5.

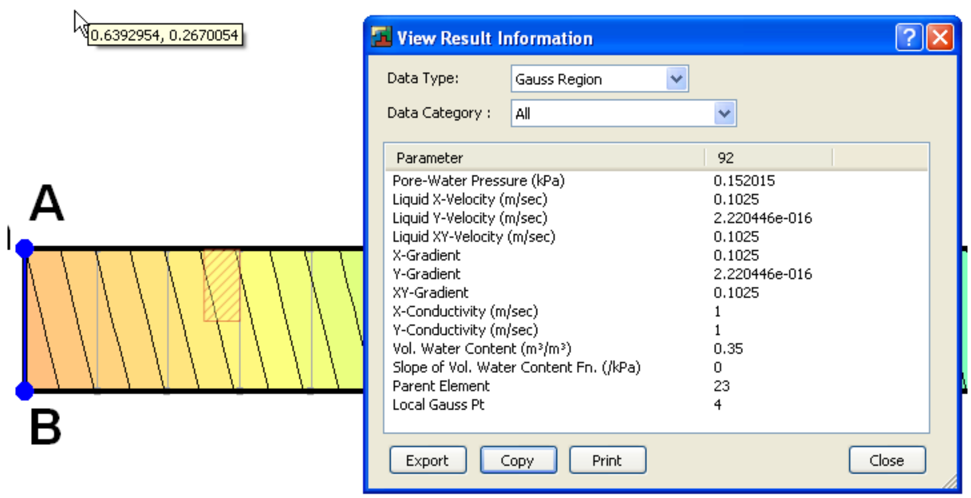


**Figure 6 Results for a Gauss Region**

## 5 Concluding Remarks

This example illustrates the fresh water hydraulic heads and fluxes calculated by SEEP/W for a coupled density-dependent groundwater flow problem. All hydraulic heads are converted to an equivalent fresh water total head based on the density of the pore fluid. Pore fluid density is a function of the solute concentration, so the solution is coupled and therefore SEEP/W must be integrated with CTRAN/W. Interpreting the computed hydraulic heads and groundwater fluxes requires an understanding of the underlying theory for density-dependent groundwater flow problems.

This simple example demonstrates that SEEP/W and CTRAN/W have been correctly coded for density-dependant flow and that the procedure of passing velocity and concentration between the two programs is an acceptable procedure for solving this coupled process, even though the two partial differential equations are not solved simultaneously.

# 6 References

Fetter, 1994. Applied Hydrogeology, 3rd Edition. Prentice Hall, Englewood Cliffs, NJ.

Frind, 1982a. Simulation of long-term transient density-dependent transport in groundwater. Advances in Water Resources, Vol. 5, pp. 73-88

Frind, 1982b. Seawater instruction in continuous coastal aquifer-aquitard systems. Advances in Water Resources, Vol. 5, pp89-97.