Probability - James Bay Case History

1 Introduction

This article looks at the SLOPE/W probabilistic analysis capabilities relative to a published case history.

The James Bay hydroelectric project in Northern Quebec, Canada, required the construction of approximately fifty kilometers of dykes on soft and sensitive clay. Divergent views were prevalent regarding the selection of appropriate factors of safety and the selection of strength properties. Due to the complexity of the geotechnical problems, an international committee was formed to address and resolve the issues (Christian et al. 1994). The issues were studied and analyzed extensively and the project has consequently become an important and often-cited case history.

The uncertainties and spatial variability in the soil properties have been documented by Ladd (1983 and 1991). Christian et al. (1994) used this data for doing a probabilistic stability analysis. El-Ramly (2001) also used this case history in his Ph.D. dissertation studies at the University of Alberta, and highlights of his work have been published by El-Ramly, Morgenstern and Cruden (2002).

SLOPE/W follows some of the concepts presented by El-Ramly et al. (2002) and it is therefore of interest to determine how the SLOPE/W implementation compares with the findings in this publication.

2 Problem description

The proposed design frequently cited in publications is presented in Figure 1. The embankment height is 12 m with 3:1 side slopes and a 56 m wide berm at mid-height

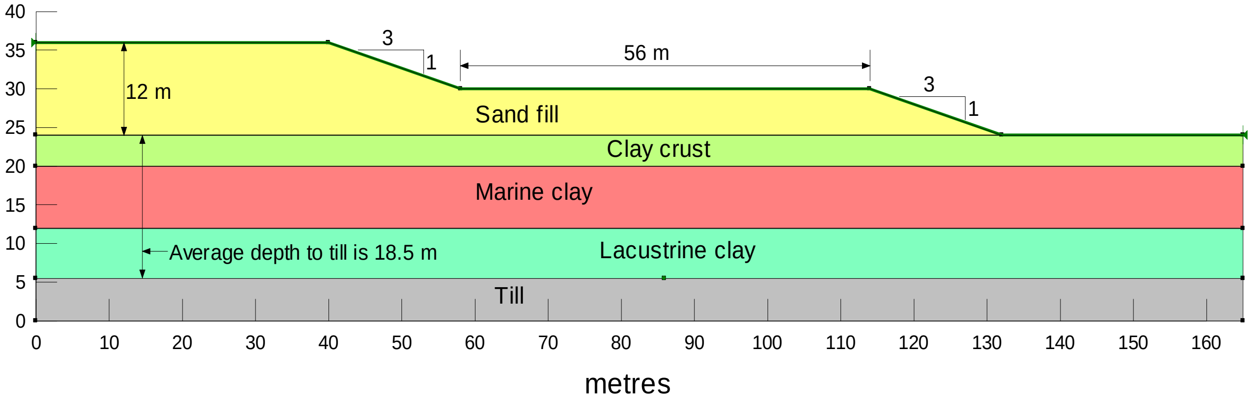

Figure 1 James Bay configuration for average conditions

On average, the first 4 m below the ground surface is a clay crust. Below this surface layer is a marine clay layer 8 m thick, and below this is a stratum of lacustrine clay. The lacustrine clay thickness varies depending on the depth to the till. The average depth to the till is 18.5 m, making the lacustrine clay about 6.5 m thick. The underlying till strength is high relative to the clays and, consequently, does not come into play in the stability analysis.

The main controlling issue here is the strength of the soft marine and lacustrine clays. The strength was measured at 1-m depth intervals at numerous locations by field vane test. Of concern was the large scatter in the strength measurements. The mean undrained strength of the marine clay was found to be about 35 kPa and for the lacustrine clay about 31 kPa.

3 Data dispersion

The dispersion in the relevant data has been quantified by Ladd et al. (1983) and Christian et al. (1994). They considered there to be eight parameters with variability that should be considered in the stability analysis. They are the:

1. Unit weight of the embankment sand
2. Friction angle of the embankment sand
3. Thickness of the clay crust
4. Undrained strength of the marine clay
5. Vane test correction factor for the marine clay
6. Undrained strength of the lacustrine clay
7. Vane test correction factor for the lacustrine clay
8. Depth to the till below the ground surface

Table 1 gives the mean and standard deviations for each of these variables.

Table 1 Mean and standard deviation values

Parameter	**Mean**	**Standard Deviation**
Unit weigh of sand kN/m^3	20.0	1.0
Friction angle of sand	30.0	1.0
Thickness of clay crust - m	4.0	0.48
Strength of marine clay – kPa	34.5	8.14
Vane correction for marine clay	1.0	0.075
Strength of lacustrine clay – kPa	31.2	8.65
Vane correction for lacustrine clay	1.0	0.15
Depth to till – m	18.5	1.0

All the variables are assumed to have a Normal distribution. The distributions are truncated at ± 3 standard deviations for the crust thickness, the depth to the till and the marine and lacustrine clay undrained strengths. This is necessary to avoid negative strength values and overlapping or elimination of the soil layers. The truncation is the same has adopted by El-Ramly et al. (2002).

Examples of the probability distributions as used in SLOPE/W are shown in Figure 2 and Figure 3.

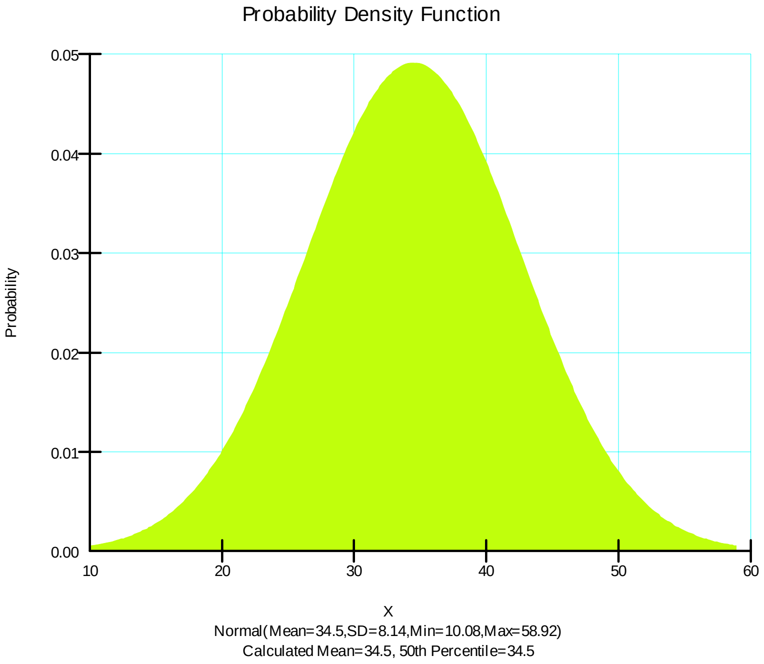

Figure 2 Probability distribution for the marine clay

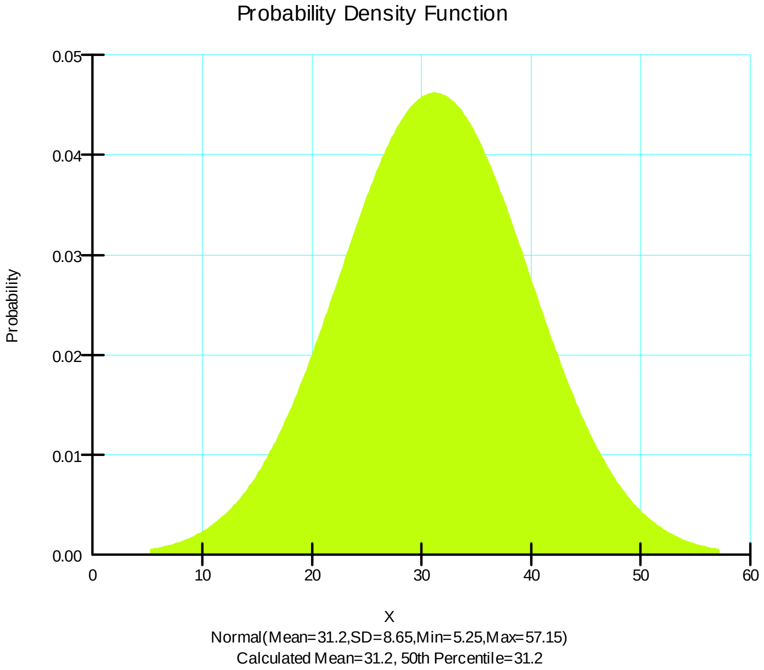

Figure 3 Probability distribution of the lacustrine clay

The current version of SLOPE/W cannot accommodate all the above statistical parameters. The parameters that cannot be considered formally in the analysis are the variable crust depth, the variable depth to the till and the variability in the vane correction factors. These uncertainties can be considered indirectly, as discussed later.

The strength of the clay crust is considered to be a constant 43 kPa. Other constant soil properties are the unit weights of foundation clays. They are 18.8, 18.8 and 20.3 kN/m^3 for the crust, marine and lacustrine clays respectively (Christian el al. 1994).

4 Critical failure mode

The previous studies by Christian el al. (1994) and El-Ramly et al. (2002) consider that the critical mode of failure is a circular slip surface that extends down to the till surface as illustrated in Figure 4. Since their analyses considered the variability in the depth to the till, their probability results include various slip surface positions.

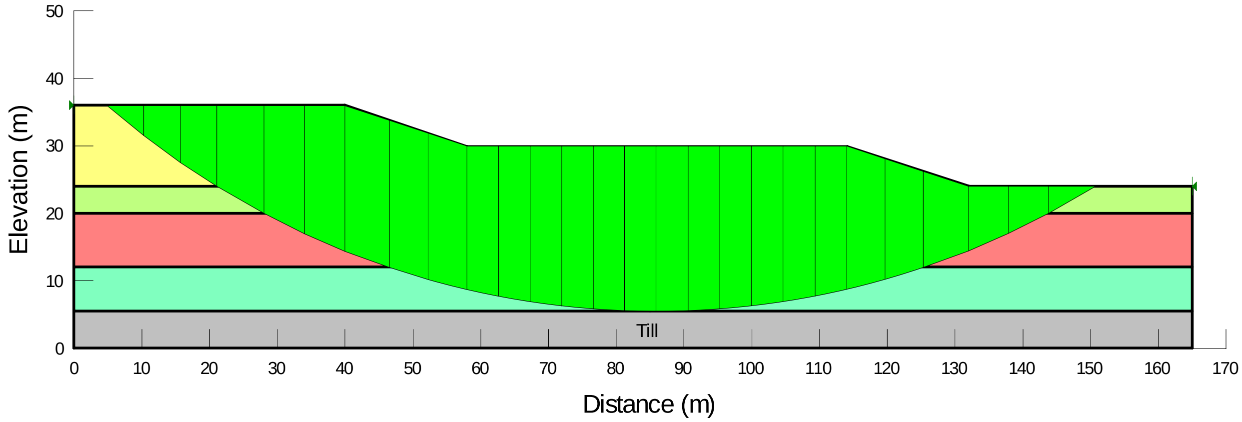

Figure 4 Shape and position of critical slip surface

5 Deterministic factor of safety

The SLOPE/W deterministic Bishop Factor of Safety for the slip surface shown in Figure 4 is 1.459. This is the same as computed by others in previous studies. Christian el al. (1994) computed a value of 1.453 and El-Ramly et al. (2002) computed a value of 1.46 using a simple spreadsheet formulation.

6 Spatial variability

Spatial variability in the soil properties can be an important consideration in certain cases. This is particularly true when the potential slip surface is relatively long within one soil, as at the James Bay site. The length of the slip surface within the lacustrine clay is over 80 m, and the slip surface passes through the marine clay at two locations more than 110 m apart.

The Monte Carlo scheme in SLOPE/W involves numerically sampling the statistical variables numerous times. The sampling can be controlled by optionally excluding or including spatial variability in the analysis. If spatial variability is included, the sampling is controlled by changes in material types and a user-specified distance. In a limit equilibrium analysis where the discretization is in the form of slices, it is convenient to view and include the spatial variability in terms of slices since it is the strength at the mid-point of each slice base that is used in the factor of safety calculations.

The spatial variability options and procedures in SLOPE/W are discussed here in terms of the slices in the potential sliding mass. Consider the diagram in Figure 5. There are a total of 27 slices. Slices 1, 2 and 3 are in the embankment fill. Slices 4 and 27 are in the clay crust. Slices 5, 6 and 7 are in the marine clay on the left and Slices 24, 25 and 27 are in the marine clay on the right. Slices 8 to 23 inclusive are in the lacustrine clay.

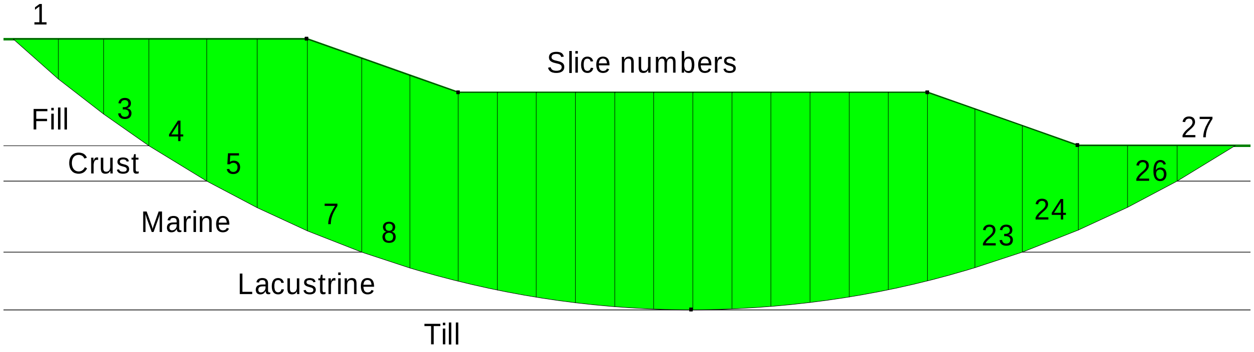

Figure 5 Slice numbers in the slide mass discretization

6.1 No spatial variability

When spatial variability is not selected as a consideration in SLOPE/W, all the statistical parameters are sampled before computing the factor of safety and the parameters are consequential constant for each Monte Carlo run. Figure 6 shows the strength (cohesion) in the clays for three Monte Carlo runs.

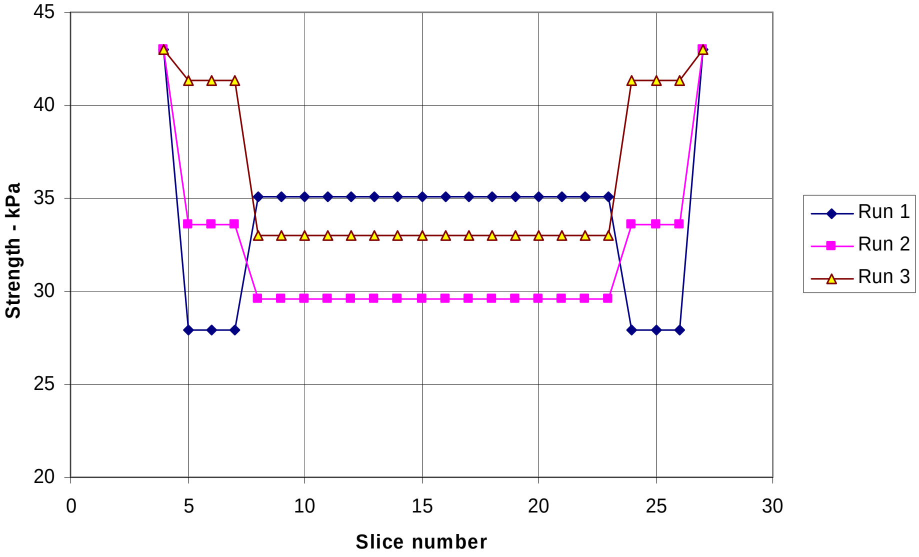

Figure 6 Strength variation along slip surface with no spatial considerations

The strength of the clay crust has no assigned probability distribution and, as a result, the strength is a constant 43 kPa (first and last data points). The strengths of the marine and lacustrine clays are different for each Monte Carlo run, but are constant for each run. Note that the marine clay strength is a constant within one run, even though the two slip surface segments within this material are far apart.

Worth noting is that for 30,000 Monte Carlo runs, there will be 30,000 different strength profiles such as in Figure 6.

El-Ramly et al. (2002) refer to this type of approach as a “simplified analysis”.

6.2 Spatial variability option

When the spatial variability option in SLOPE/W is selected, the statistical parameters are sampled, (1) each time the slip surface enters a different material, and (2) when the distance within one material exceeds a user-specified sampling distance.

Consider the case when the sampling distance is specified as 30 m for the slip surface shown in Figure 5. The first and last data points again represent the constant clay crust strength. Now, however, the marine clay strength is different in the left and right slip surface segments within this material. For Run 1, the strength at the left is 44.5 and at the right the strength is 28.5 kPa.

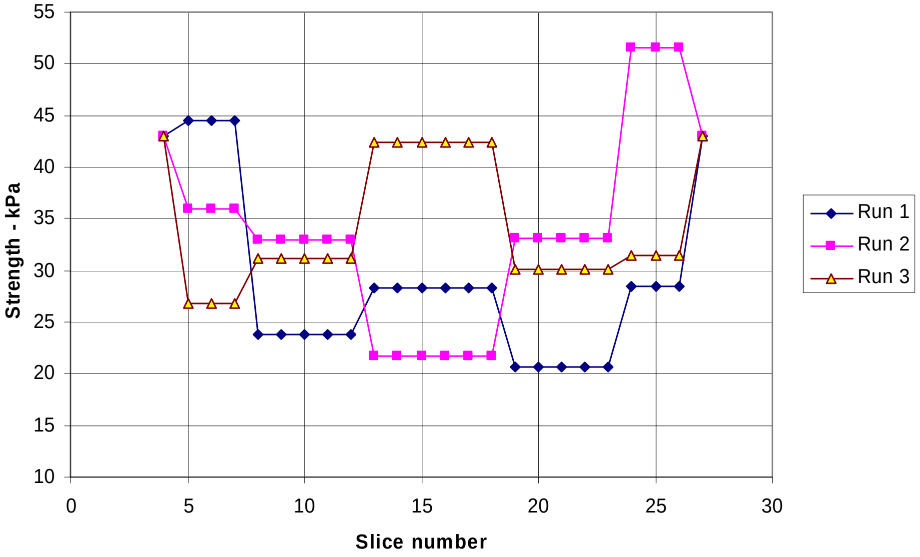

Figure 7 Variation in the clay strengths with a specified sampling distance of 30 m

The length of the slip surface within the lacustrine clay is about 80 m. The 30 m sampling distance consequently results in three different strengths along the slip surface within this material as is evident in Figure 7.

SLOPE/W tracks the distance along the slip surface and when the distance exceeds the specified sampling distance, the properties are sampled again; that is, the dice, so to speak, are rolled again to get new material properties.

When the last sampling segment along the slip surface within one soil is shorter than the specified distance, the strength is correlated with the previous segment as described by Krahn (2004). The procedure involves local averaging as described by El-Ramly et al. (2002).

In SLOPE/W the properties are sampled only once for a particular soil if the specified sampling distance is greater than the actual length of the slip surface within that soil. For the case under consideration here, the lacustrine clay strength would be a constant for each Monte Carlo run if the sampling distance is, for example, specified as 100 m. However, the clay strengths of the marine clay would be different for the left and right slip surface segments, since the properties are sampled each time the slip surface passes into a different material.

6.3 Embankment unit weight variability

Properties such as the unit weight are sampled each time the slip surface passes into a different material or each time the soil strength is sampled within one material, when the spatial variation option is selected. Figure 8 illustrates the variation in the unit weight of the embankment material when the sampling distance is specified as 30 m.

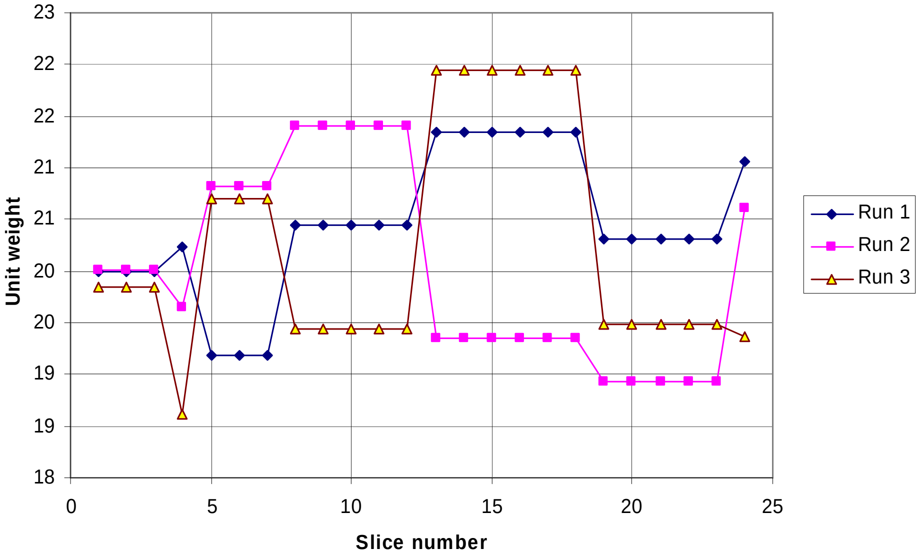

Figure 8 Variation in the embankment unit weight

When the spatial variation option is not selected, the unit weight is sampled once for each Monte Carlo run.

6.4 Sampling every slice

SLOPE/W has an option that samples the statistical parameters for every slice. Figure 9 shows the variability in the clay strengths when this option is selected.

When the properties are sampled too often for one Monte Carlo run, the solution tends towards the mean or overall average conditions. This consequently exhibits the lowest probability of failure. Moreover, the results are probably not all that realistic.

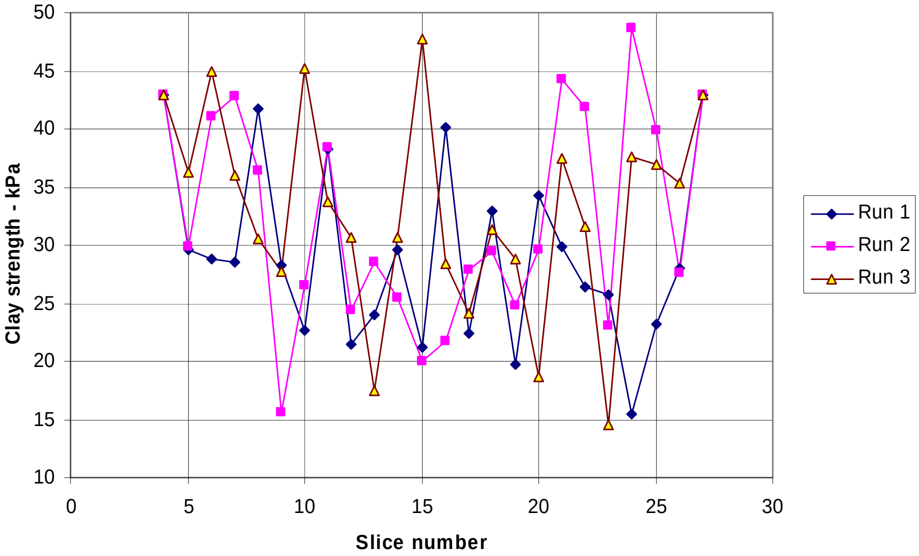

Figure 9 Strength variation in the clays when sampled every slice

Sampling the statistical parameters too much can become rather meaningless. It is somewhat analogous to creating a probability density histogram. If the interval becomes too small, it is difficult to detect a distribution of any quantifiable shape. The same problem exits if the interval is too high. Another analogous example is the descriptions of rainfall rates. Rainfall rates per second or per minute are somewhat meaningless, except perhaps under very extreme conditions. On the other hand, rainfall rates per month are often also not a good measure of rainfall intensity. Such a measure is good for describing quantities of rainfall for a period, but not the intensity.

Sampling the statistical properties for every slice does not likely give a realistic picture of uncertainty, as already note above. The feature is nonetheless available in SLOPE/W as a convenient way of bracketing the lower end of a range in probability of failure. Allowing for no spatial variability tends to bracket the upper end of the range. This is discussed further below.

7   Effect of spatial variability on probability of failure

Table 2 presents the probability distribution characteristics relative to spatial variability. The highest probability of failure occurs when no consideration is given to spatial variability (1.44%). This is logical, since it is possible that all the statistical parameters have their lowest values all at the same time. This also results in the lowest possible factor of safety.

The lowest probability of failure occurs when sampling the statistical parameters for every slice. In fact, the lowest of the 30,000 computed factors of safety is 1.227, and since this value is greater than 1.0, the probability of failure is nonexistent. Of interest though is that the lowest factor of safety is less than the deterministic factor of safety, which is based on mean values (1.227 versus 1.460).

There is a ten-fold difference in the probability of failure between 30-m and 50-m sampling distances for example. This seems like a big difference, but care is required in interpreting such difference. The fact that all probabilities of failure are less than 1.0 % when some form of spatial variability is considered may be a more important observation than the absolute values. More importantly, perhaps, is the issue in the difference between none versus some spatial variability, rather than the actual sampling distance.

Table 2 Probability of failure as a function of sampling distance

Sampling	Mean F of S	Standard Deviation	Probability of Failure (%)	Reliability Index	Min F of S
Every slice	1.461	0.065	0.00	7.118	1.227
30 m	1.460	0.128	0.01	3.599	0.960
40 m	1.460	0.146	0.04	3.152	0.913
50 m	1.458	0.156	0.11	2.946	0.896
80 m	1.458	0.197	0.92	2.321	0.765
100 m	1.460	0.198	0.96	2.321	0.761
No spatial Considerations	1.463	0.214	1.44	2.159	0.726

The issue of selecting appropriate sampling distances is discussed further below.

8 Vane strength corrections

Christian et al. (1994) and El-Ramly et al. (2002) included a vane strength correction as one of the statistical parameters for the marine clay and the lacustrine clay. The current version of SLOPE/W does not allow for a modification distribution function to be applied to a soil property distribution function. The vane corrections could, however, be included in an approximate way by correcting the field measurements before computing the mean and standard deviation. The correction would then be included indirectly in the soil property distribution function. Presumably, the mean and standard deviation would be different for the corrected and uncorrected data sets.

9 Effect of geometric variability

Christian et al. (1994) and El-Ramly et al. (2002) also included variations in the clay crust thickness and in the depth to the till. Varying the depth to the till alters the thickness of the lacustrine clay and the position of the slip surface. This has a significant effect, because such a large portion of the slip surface is within the lacustrine clay. A deeper slip surface means a larger portion of the slip surface length is in lacustrine clay, and since this is the weakest material, it significantly influences the stability.

The variable soil thicknesses cannot be considered statistically in the current version of SLOPE/W, but the effect can be considered by looking at various configurations and potential slip surfaces. The average thickness of the clay crust is 4 m with a standard deviation of about 0.5 m. If we look at a truncation of three standard deviations (as for the clay properties) as a limit, the clay crust could be as thin as 2.5 m. The standard deviation on the depth to the till is 1.0 m. Again with a truncation of three standard deviations as a limit, the depth to the till could be as much as 21.50 m. With these clay thicknesses, the James Bay configuration is as in Figure 10. Figure 11 shows the position of the slip surface within this configuration.

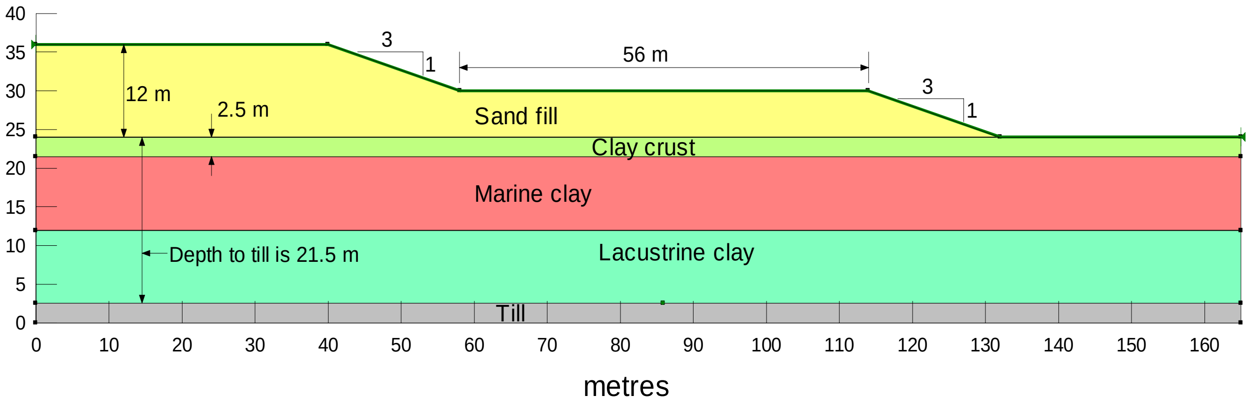

Figure 10 Configuration with thin crust and thick lacustrine clay

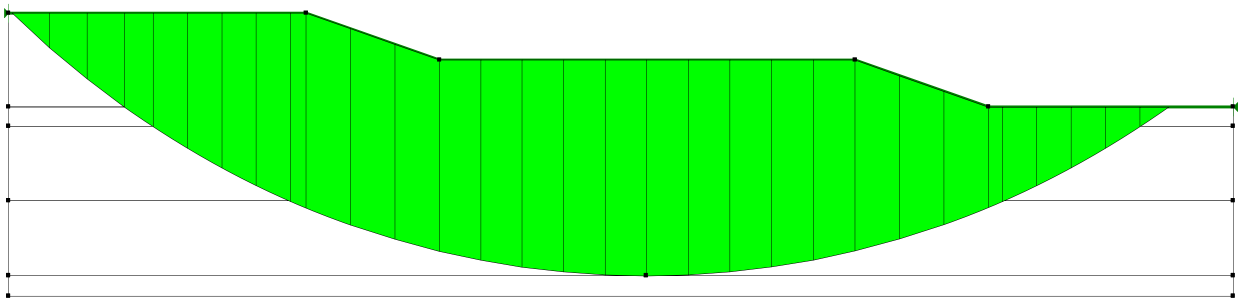

Figure 11 Position of slip surface with thin crust and thick lacustrine clay

The deterministic factor of safety now is 1.310, about 15 % lower than for the average conditions.

Table 3 shows the probabilistic parameters for the altered geometry. The probabilities of failure are now significantly higher. At a 30-m sampling distance, for example, the increase is from 0.01% up to 0.48% and when consideration is given to spatial variation, the increase is from 1.44% up to 7.58%

Table 3 Probability parameters with thin crust and thick lacustrine clay

Sampling	**Mean F of S**	**Standard Deviation**	**Probability of Failure (%)**	**Reliability Index**	**Min F of S**
Every slice	1.311	0.058	0.00	5.347	1.093
30 m	1.310	0.122	0.48	2.540	0.810
40 m	1.310	0.136	1.00	2.280	0.800
50 m	1.310	0.153	2.12	2.024	0.749
80 m	1.309	0.183	4.57	1.689	0.708
100 m	1.310	0.206	6.60	1.509	0.595
No spatial Considerations	1.313	0.219	7.58	1.431	0.555

These increases in the probabilities of failure are a direct reflection of the lower deterministic factor of safety, or a direct reflection of a more critical mode of failure. As discussed further below, this follows a usual trend that the higher probabilities of failure occur for the deterministically determined most critical slip mode.

10 Comparison with published analyses

It is difficult to make a direct comparison with the results presented by others, because the details are different in each study. The closest is the study by El-Ramly et al. (2002). SLOPE/W follows the sampling distance concepts presented in this study. El-Ramly et al. used three sampling segments in the lacustrine clay, two in the marine clay and four in the embankment. They included the depth to the till as a variable and, since this geometric variation significantly increases the probability of failure, the closest condition here to the El-Ramly analysis is the one with the thick lacustrine clay with a 30-m sampling distance. For this condition, the comparison between these two analyses is as listed in Table 4.

Table 4 Comparison of different analyses

	SLOPE/W analysis	**El-Ramly Analysis**	**Christian Analysis**
Probability of failure (%)	0.48	0.47	
Reliability index	2.54	2.32	2.66

Christian el al. (1994) did a FOSM (first-order second moment) type of analysis. Their reliability index for a comparable situation is 2.66, as shown in Table 4 (they do not present their results in terms of probabilities of failure).

In spite of the differences in all three studies, the findings are remarkably close, and this demonstrates that the SLOPE/W formulation gives results comparable with other different and independent analyses.

11 Closing remarks

The James Bay probabilistic studies have shown the important effect of including spatial variability in an analysis. It is particularly important for this case because of the relatively long slip surface, especially in the lacustrine clay. It seems unreasonable to think that this clay would have the lowest strength predicted from the probability distribution over the entire distance of the slip surface, especially considering the large scatter in the field measurements.

If sufficient data is available, a variogram can be determined, as discussed by Soulié et al. (1990). The form of a variogram is illustrated in Figure 12. The differences in strength, for example, increase with distance between sampling points up to a certain distance, after which the difference remains constant. The point at which the variogram becomes horizontal is called the range, and it is the distance beyond which there is no correlation between strength measurements. The range from a variogram would be an appropriate sampling distance for a SLOPE/W probabilistic analysis.

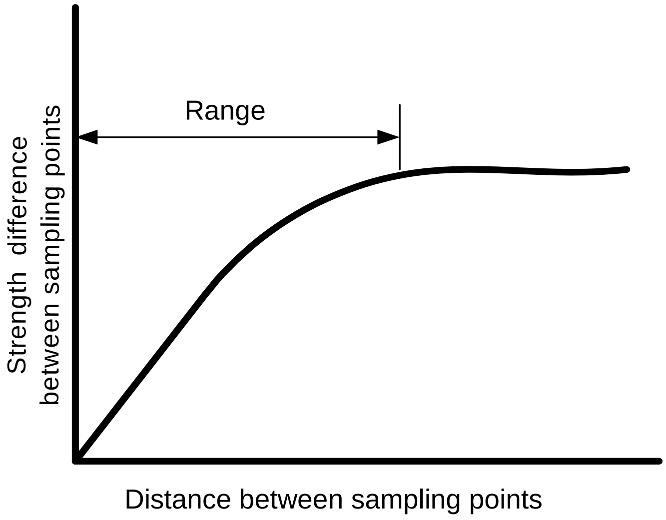

Figure 12   A typical variogram

Unfortunately, sufficient data is seldom available to formally evaluate the spatial variability, and so it comes down to making a judgment. Making an intuitive judgment, however, may be better in some cases than ignoring the possibility of spatial variability completely. El-Ramly et al. (2002) go so far as to say that ignoring spatial variability, "… can be erroneous and misleading." This may be overstating the case for some analyses, but it does highlight the importance of giving spatial variability its due consideration in an analysis. The beauty of a tool such as SLOPE/W is that various sampling distances can be examined easily and quickly to assist with making a judgment on an appropriate sampling distance.

An important point to note in the Christian et al. (1994) and the El-Ramly et al. (2002) studies is that the slip surface position is tied to the till elevation. This is probably acceptable, since the undrained strengths of the clays are taken to be constant with depth, and for undrained strengths, the critical slip surface tends to go as deep as possible. In many situations the soil strengths tend to increase with depth and the position of the critical surface may then be somewhere within the soil stratum. Including geometric variability for such a situation becomes much more complicated. The best way to handle this in the current version of SLOPE/W is to do a probabilistic analysis on multiple slips surfaces. An option is available to do a probabilistic analysis on a user selected number of slip surfaces closets to the most deterministically critical slip surface.

One way of implementing geometric variably in SLOPE/W would be to assign probability distributions to point coordinates. For the James Bay case, probability distributions could, for example, be assigned to the y-coordinates of the end-points of the line representing the till surface. The radius point would then have to follow the positions of the boundary line. This is conceivable for the James Bay configuration, but allowing this generally for all geometric points could easily lead to confusion and perhaps meaningless results, so such a concept would have to be implemented with restrictions. This could be done in the future if user demand warrants it.

SLOPE/W perhaps does not accommodate all the nuances and refinements possible in a probabilistic stability analysis. However, the extensive range of features and capabilities available are likely adequate for use in practice, especially considering that probabilistic analyses are as yet not routine in practice. It is considered important too not over-complicate the issues when firm practice approaches have not yet been established. Complexity and capability can and will be added as probabilistic analyses mature more in practice.

The big attraction of a tool such as SLOPE/W is the ease with which probabilistic analyses can be performed. Once a problem has been set up for a deterministic analysis, there is very little extra modeling effort involved in doing a probabilistic analysis. No extra tools are required. A variety of probabilistic distribution functions are available with truncation as necessary. Even a general data point distribution function is available for any unusual distribution of properties. The most difficult part of the analysis is obtaining sufficient data to define the dispersion appropriately.

The ability of plotting parameters like strength along the slip surface for individual Monte Carlo runs has as yet not been implemented in SLOPE/W. The plots in Figures 6 to 9 were extracted from the code specifically for this presentation. to illustrate the techniques and procedures. This is another feature that could potentially be included in the future.

12 References

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