Probability and Sensitivity

1 Introduction

The purpose of this illustrative example is to show how you can do a probabilistic analysis and a sensitivity analysis of the stability of a slope. Features of this simulation include:

- Analysis method: Morgenstern-Price with Half-sine function
- Statistical variability in some parameters
- Multiple soil layers with different soil models
- Pore water pressure with piezometric line
- Entry and Exit slip surface with projection angle
- Auto search for tension crack zone

2 Example Problem Definition and Material Properties

Figure 2 shows the geometry, the piezometric line and the entry and exit search zones. The red arrows on the entry zone indicate the projection angle of the slip surfaces.

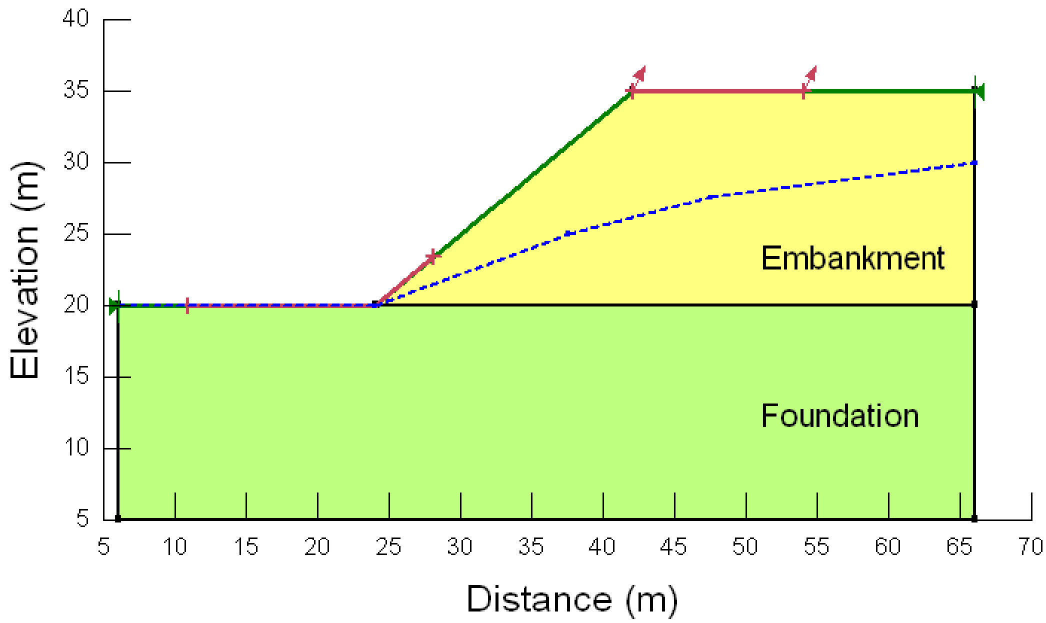

Figure 2 Geometry and Piezometric line of the example

The embankment is modeled with a Mohr-Coulomb soil model. The unit weight is 20 kN/m^3, c' is 30 kPa and ϕ' is 40^o. The foundation material is modeled with a S=f(overburden) soil model. The unit weight is 20 kN/m^3, the Tau/Sigma Ratio is assumed to be 0.45.

3 Case 1 – Without tension crack

Figure 2 shows the most critical slip surface and the computed factor of safety when there is no tension crack. The factor of safety is 1.200. Figure 4 shows the free body diagram and the force polygon of slice 31 near the crest of the slope. Note that the base normal force is negative, indicating that the base normal force is in tension. Negative base normal force is quite common for slices near the crest of the slope when the entry angle is steep and when the underlying soil is weak. The negative normal force is troublesome, since soil cannot be in tension theoretically.

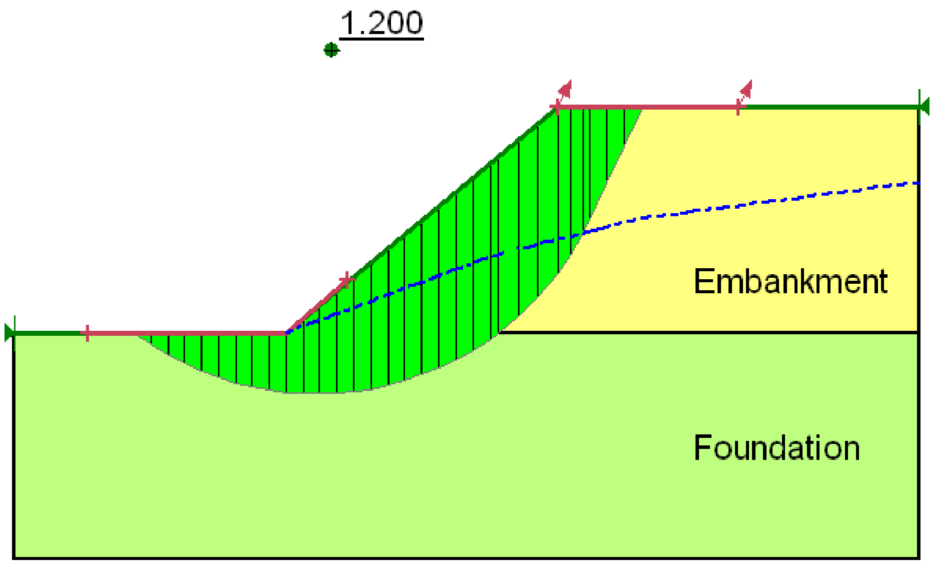

Figure 2 Factor of safety of the example PROBABILISTIC without any tension crack

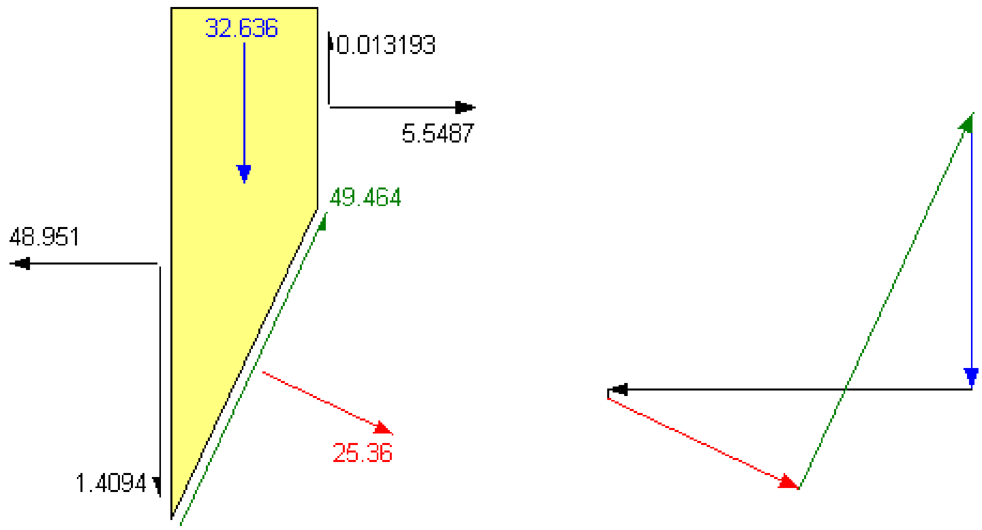

Figure 3 Free body diagram and force polygon showing negative base normal force

4 Case 2 – With tension crack

SLOPE/W has the option of automatically searching for tension crack zone. This is helpful in cases where the tension crack zone is not known. Figure 4 shows the most critical slip surface and the computed factor of safety when the option of "Search for tension crack" is selected. The factor of safety is 1.185, and a tension crack is located about 3 m below the crest. Figure 5 shows the free body diagram and the force polygon of the last slice near the crest of the slope. Note that the normal force is positive.

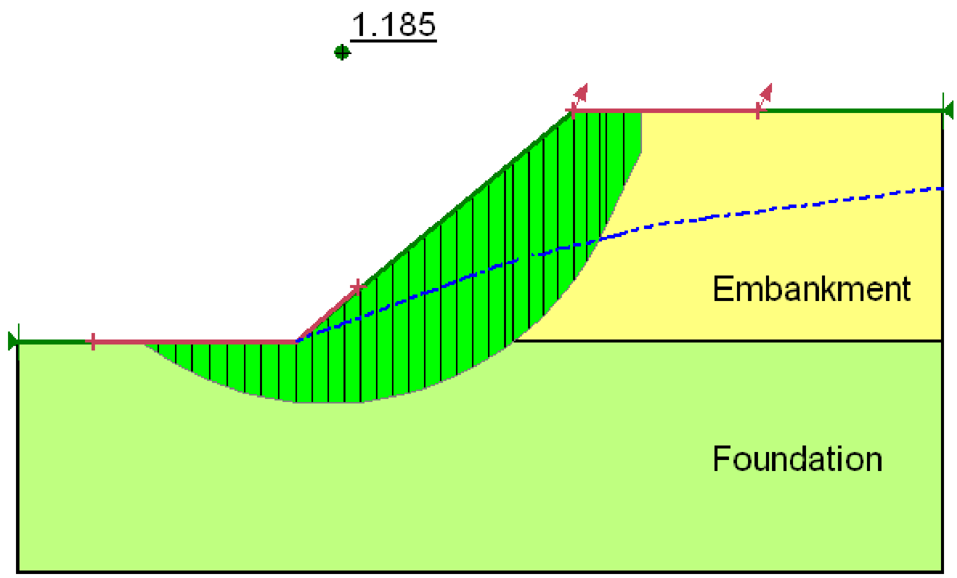

Figure 4 Factor of safety of the example PROBABILISTIC with auto search for tension crack

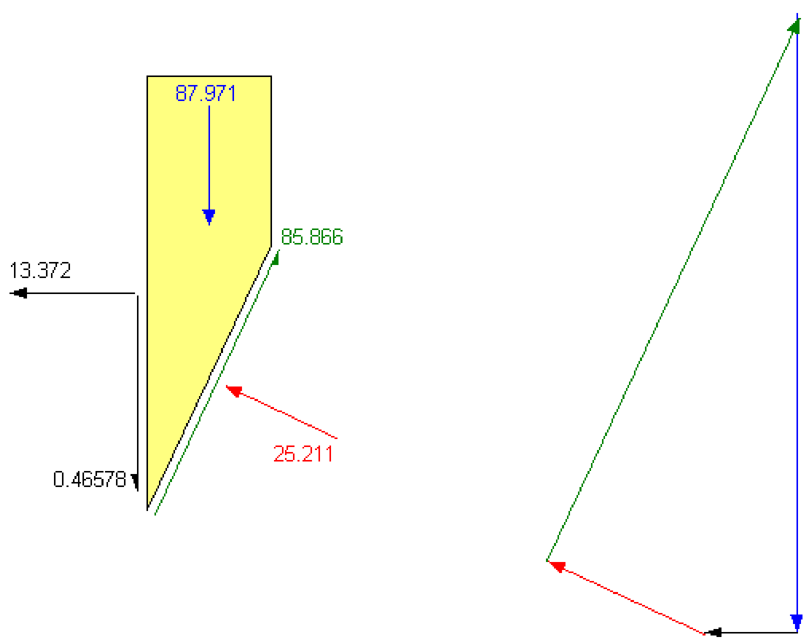

Figure 5 Free body diagram and force polygon showing positive base normal force

5 Case 3 – Probabilistic analysis

A probabilistic analysis can be performed quite easily with SLOPE/W when the variability of the soil properties or other input parameters is known. Although SLOPE/W allows various type of variability distribution, let us assume that the variability is normally distributed with a known standard deviation. In this example, the standard deviation for ϕ', c', Tau/Sigma Ratio and piezometric line is 6^{o}, 3 kPa, 0.05 and 1 m respectively. Figure 6 shows the variability in a normal distribution function.

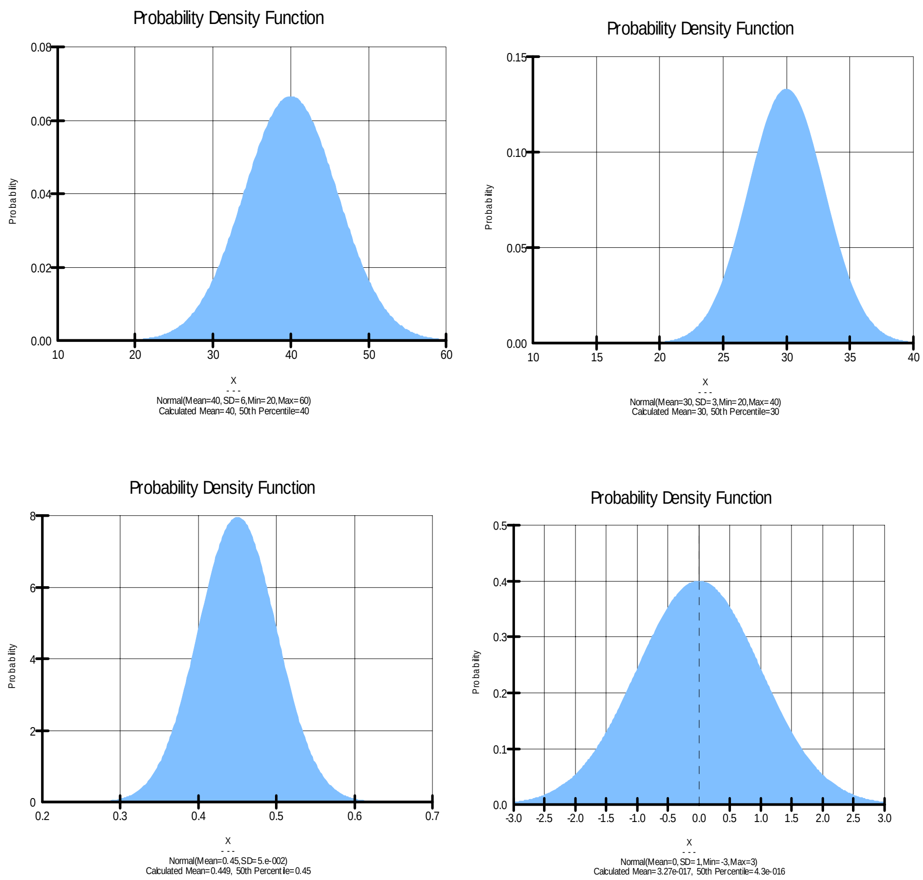

Figure 6 Normal distribution of ϕ' (upper left) , c' (upper right), Tau/Sigma Ratio (lower left) and piezometric line (lower right)

After 5000 Monte Carlo trials, the following results are obtained:

Mean F of S = 1.2042

Reliability Index = 1.437

Probability of Failure = 6.64 %

Standard deviation of F of S = 0.142

Min F of S = 0.6762

Max F of S = 1.7861

Using the “Draw Probability” feature in CONTOUR, you can get a probability density function of the 5000 computed factors of safety. Since the variability of the soil properties is normally distributed, the probability density function of the factor of safety (Figure 7) is also normally distributed as expected.

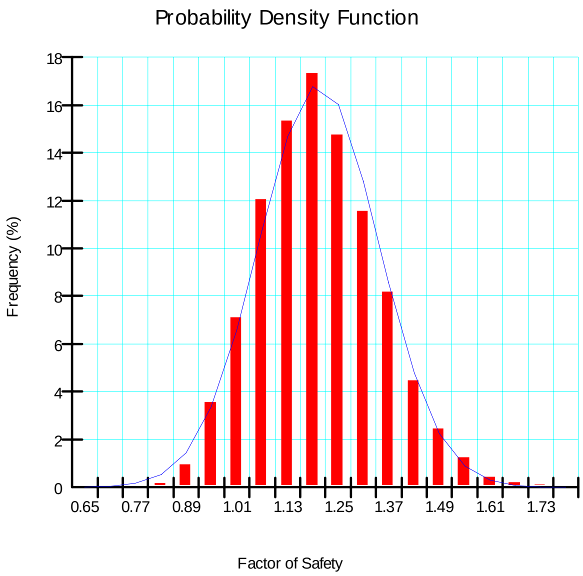

Figure 7 Probability density function of the 5000 Monte Carlo factors of safety

Figure 8 shows the probability distribution function of the 5000 Monte Carlo factors of safety. The red line represents the probability of failure.

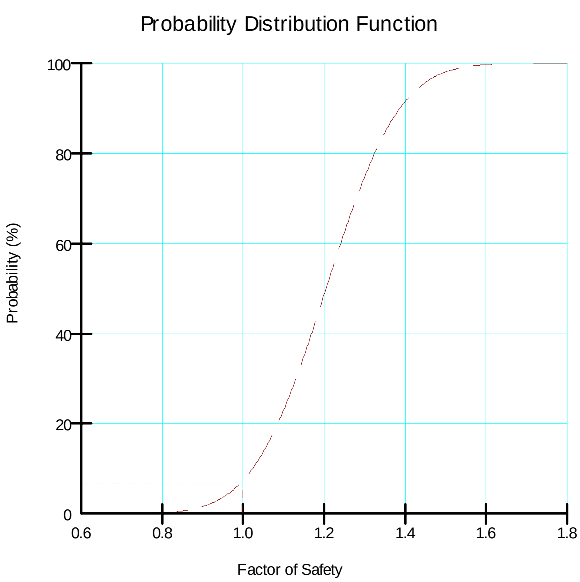

Figure 8 Probability distribution function of the 5000 Monte Carlo factors of safety showing probability of failure

In this case, for each Monte Carlo trial, the soil properties of the entire soil layer are only sampled once, the sampling value is applied to all slices within the same soil layer, the probability of failure is 6.64%. The other extreme is to sample new soil properties for every slice in each Monte Carlo trial. The probability of failure is now reduced to 0.78%. These two options are useful to compute the range of the probability of failure. The more realistic probability of failure is likely in between of 0.78% and 6.64%. When more field investigation and soil testing are available, and when more statistical data become available to justify the use of a certain sampling distance, the probability of failure can be more accurately estimated. For example, if you assume a sampling distance of 10 m, you will get a probability of failure of 3.36%. To evaluate what should be the correct sampling distance in a probabilistic analysis, you will need a variogram analysis (Figure 9), which is beyond the scope of this discussion.

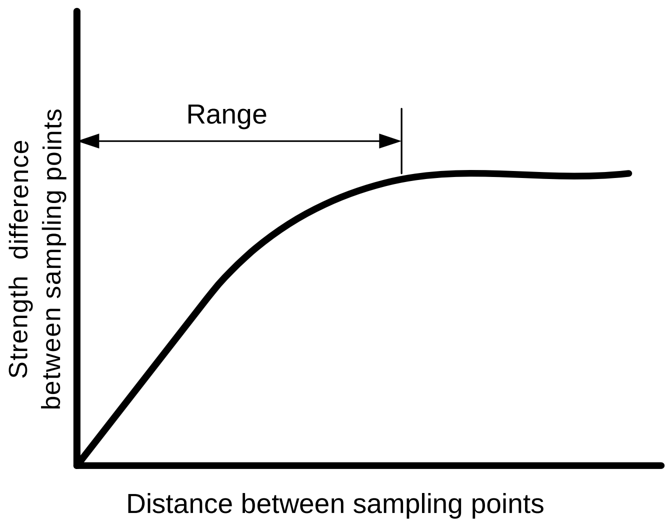

Figure 1   A typical variogram showing the Range (Theoretical sampling distance)

Unfortunately, sufficient data is seldom available to formally evaluate the spatial variability, and so it comes down to making a judgment. Making an intuitive judgment, however, may be better in some cases than ignoring the possibility of spatial variability completely. El-Ramly et al. (2002) go so far as to say that ignoring spatial variability, "… can be erroneous and misleading." This may be overstating the case for some analyses, but it does highlight the importance of giving spatial variability its due consideration in an analysis. The beauty of a tool such as SLOPE/W is that various sampling distances can be examined easily and quickly to assist with making a judgment on an appropriate sampling distance.

SLOPE/W perhaps does not accommodate all the nuances and refinements possible in a probabilistic stability analysis. However, the extensive range of features and capabilities available are likely adequate for use in practice, especially considering that probabilistic analyses are yet not routine in practice. It is considered important to not over-complicate the issues when firm practice approaches have not yet been established. Complexity and capability can and will be added, as probabilistic analyses mature more in practice.

The big attraction of a tool such as SLOPE/W is the ease with which probabilistic analyses can be performed. Once a problem has been set up for a deterministic analysis, there is very little extra modeling effort involved in doing a probabilistic analysis. No extra tools are required. A variety of probabilistic distribution functions are available with truncation as necessary. Even a general data point distribution function is available for any unusual distribution of properties. The most difficult part of the analysis is obtaining sufficient data to define the dispersion appropriately.

6 Case 3 – Sensitivity analysis

SLOPE/W allows you to specify a value range of the material parameters, and will compute the factor of safety automatically when each value of the parameters is used. Let's assume that you are interested in the following soil properties ranges:

- Unit weight - mean value at 20, study from 17.5 to 22.5 with 10 increments
- Cohesion – mean value at 30, study from 15 to 45 with 10 increments
- Friction angle – mean value at 40, study from 35 to 45 with 10 increments

You can do a sensitivity study of the above. At the end, SLOPE/W provides a sensitivity graph (Figure 10) showing the computed factor of safety at different values of the parameters. For example, you can see that the factor of safety is most sensitive to the cohesion of the embankment, and is least sensitive to the material weight. The crossing point represents the factor of safety (1.185) when the mean value of all the parameters is used. SLOPE/W also provides a table listing the value of a parameter used and the respective factor of safety computed (Figure 11).

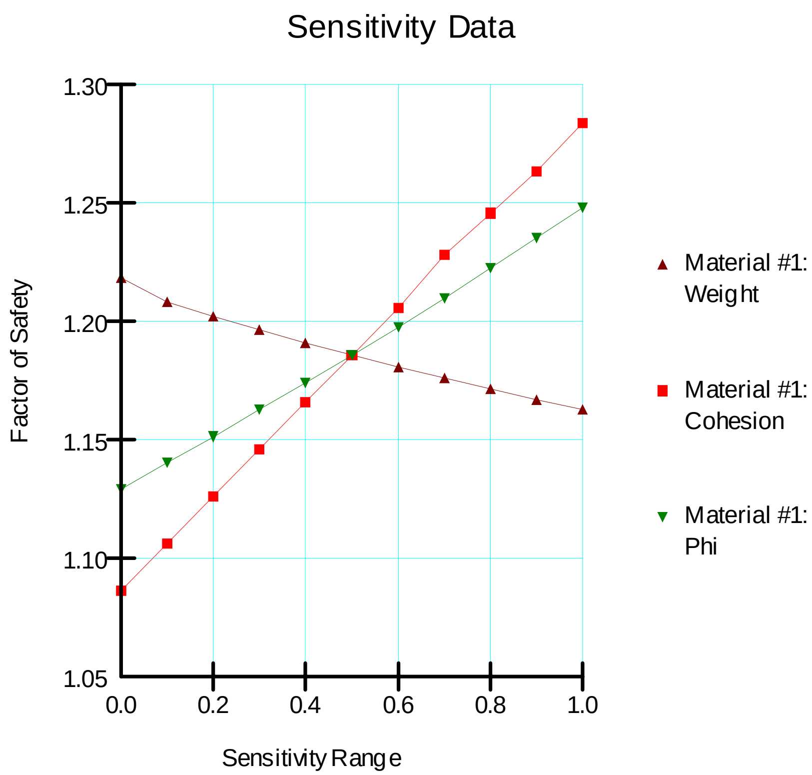

Figure 10 A plot showing the sensitivity of the computed factor of safety versus the range of the parameters used in the computation

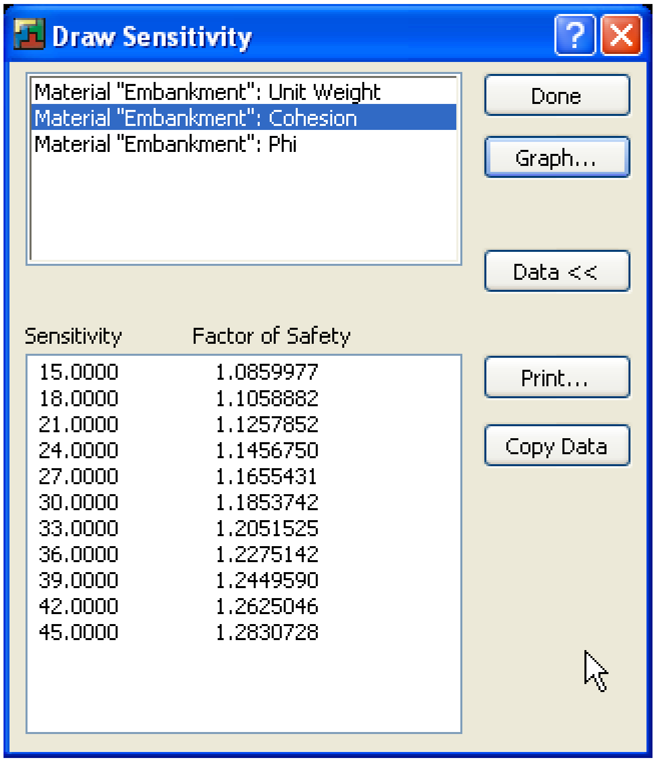

Figure 11   A data table showing the value of the parameter and the computed factor of safety

7 References

El-Ramly, H., Morgenstern, N.R. and Cruden, D.M. (2002). *Probabilistic Slope Stability Analysis for Practice*, Canadian Geotechnical Journal, Vol. 39, No. 3, pp. 665-683.