

Verification of the Hyperbolic Soil Model by Triaxial Test Simulations

1 Introduction

This example simulates a series of triaxial tests that can be used to verify that the Hyperbolic constitutive model is functioning properly. The simulations include:

- Consolidating the sample to an initial isotropic stress state
- A drained strain-controlled test
- A load-unload-reload test
- An extension test
- Consolidation at anisotropic stress conditions
- Consolidation with a back-pressure

The verification includes comparisons with hand-calculated values and discussions relative to the Hyperbolic theoretical framework.

2 Feature highlights

GeoStudio feature highlights include:

- Using the axisymmetric option to simulate a triaxial test
- Displacement type boundary conditions to simulate a strain-controlled test
- A displacement boundary function to simulate loading, unloading, and then reloading
- Use of total stress and effective stress soil parameters
- Application of two different normal stresses to simulate anisotropic consolidation

3 Included files

Full details of this example and the GeoStudio files are included as:

Hyperbolic triaxial tests.gsz and *Hyperbolic triaxial tests.pdf*

The specifics of each analysis are available in the GeoStudio data file.

4 Hyperbolic constitutive model

The hyperbolic constitutive model is, in essence, a linear-elastic formulation, but the Young's Modulus E_t is a function of the shear stress level in the soil as described by:

$$E_t = \left[1 - \frac{R_f}{2c} \frac{\sigma_1 - \sigma_3}{\cos \phi + 2\sigma_3 \sin \phi} \right]^2 E_i$$

- E_i is the stiffness modulus when the soil is in an isotropic state; that is, there is no shear stress

- ϕ is the soil friction angle
- C is the soil cohesion
- R_f is a parameter controlling how close the stress-strain curve comes to the shear strength

The term $\frac{\sigma_1 - \sigma_3}{2} (1 - \sin \phi)$ represents the shear stress level

The term $2c \cos \phi + 2\sigma_3 \sin \phi$ represents the shear strength of the soil

Therefore when $\frac{\sigma_1 - \sigma_3}{2} (1 - \sin \phi)$ divided by $2c \cos \phi + 2\sigma_3 \sin \phi$ is 1.0 or greater, it means that the shear stress is at the shear strength

If this ratio is computed as being greater than 1.0, SIGMA/W sets the ratio to 1.0

When the shear stress is at the shear strength then the tangent modulus is,

$$E_t = \left[1 - R_f \frac{\sigma_1 - \sigma_3}{2} (1 - \sin \phi) \right]^2 E_i$$

If, for example, R_f is 0.9, E_t then is 0.01 E_i or 100th of E_i when the shear stress is at the shear strength. If R_f is 0.7, then E_t is 0.09 E_i . In other words, the higher the R_f values the closer E_t is allowed to come to zero. R_f values also affect the shape of the stress-strain curve – higher R_f values result in stress-strain curves with a sharper bend than lower R_f values.

Very low E values can result in excessive and unrealistic deformation and lead to numerical instability (non-convergence).

In addition to the specified R_f value, SIGMA/W controls the minimum E_t relative to the atmospheric pressure.

SIGMA/W does not allow E_t to fall below the atmospheric pressure value

The reason for arbitrarily restricting the minimum E_t value is to reduce the possibility of encountering convergence difficulties.

E_i is a user defined constant or a user-defined function relative to the vertical effective stress in the ground.

5 Analysis 1-Cohesion only

This analysis simulates an unconfined compression test. The unconfined compressive strength C_u of the soil is 100 kPa and the initial Modulus E_i is 5000 kPa.

The sample is loaded at a constant strain rate of -0.004 m (4 mm) per load step. The negative sign indicates downward. There are 75 load steps and the results are saved every 5th step. The total strain is 30% (0.004 * 75 * 100).

The maximum vertical stress when the sample is at its shear strength should be 200 kPa.

$$\tau_{\max} = (\sigma_1 - \sigma_3) / 2$$

The horizontal stress σ_3 is zero, since this an unconfined test. τ_{\max} is a specified 100 kPa. Therefore σ_1 (vertical stress) at failure will be 200 kPa.

Figure 1 shows the SIGMA/W stress-strain curve. The final strain is 0.3 (30%) as specified for this analysis and the maximum vertical stress approaches 200 kPa.

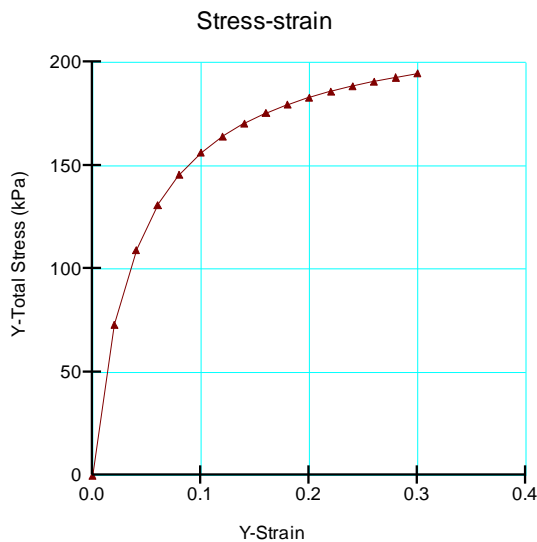


Figure 1 Unconfined compression test with $C_u = 100$ kPa

The vertical stress will continue to increase if the sample is loaded further. In fact the vertical stress will reach the theoretical maximum and then exceed the maximum. The reason for this is that the tangent modulus E_t is never allowed to go to zero.

In this analysis R_f is specified as 0.9. E_t therefore should be allowed to fall to $0.01 * E_i = 0.01 * 5000 = 50$ kPa. The atmospheric pressure has, however, been set at 100 kPa (rounded from 101.28 for discussion and presentation convenience) and the lowest E_t value consequently is 100 kPa.

Figure 2 shows the variation of E_t during the loading. E_t starts at the specified E_i of 5000 kPa and then diminishes as the shear stress increases and ends at 100 kPa.

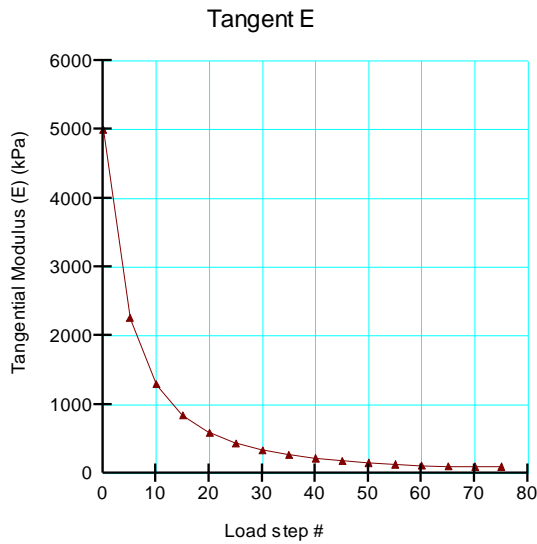


Figure 2 Variation of tangent modulus

6 Analysis 2-C only load-reload

The previous analysis is repeated here, but the sample is loaded, then unloaded and then reloaded.

The top of the sample is pushed down, pulled up and then pushed down according to the function in Figure 3.

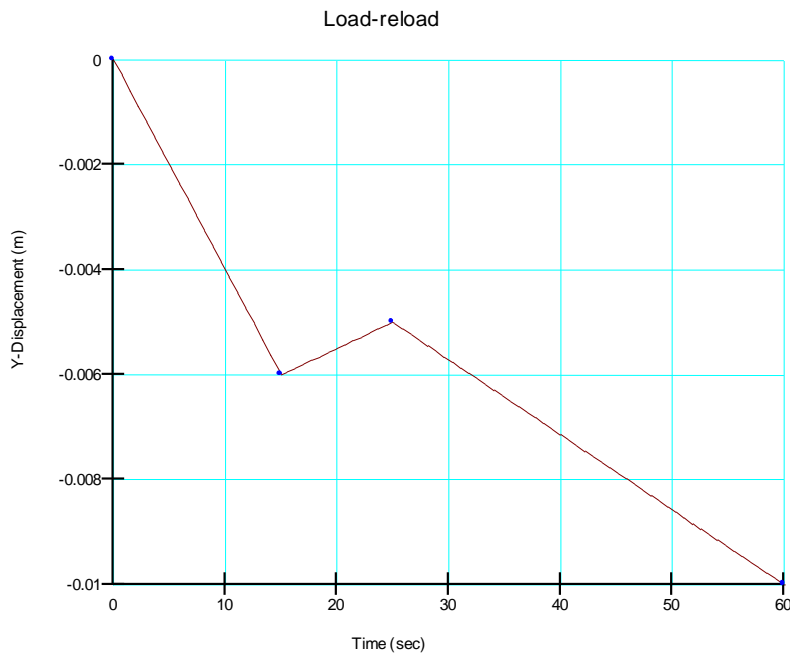


Figure 3 Loading, unloading, and reloading boundary function

Figure 4 shows the resulting stress-strain curve.

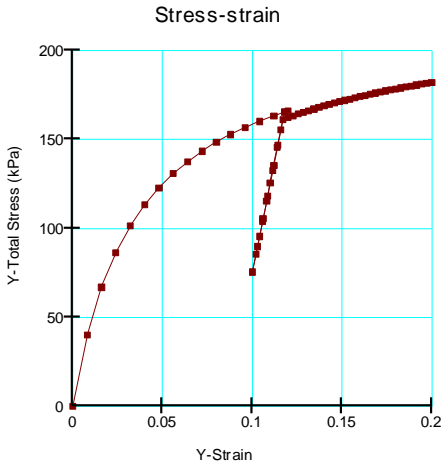


Figure 4 Stress-strain behavior during loading, unloading and reloading

During the unloading, SIGMA/W sets E_t to E_i as illustrated in Figure 5. E_t diminishes due to the shearing, then jumps up to 5000 (E_i) when the unloading starts, and then stays at 5000 kPa until the shear stress exceeds the previous maximum.

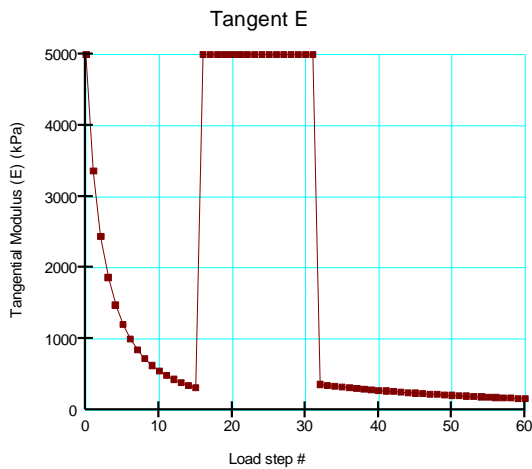


Figure 5 Variation of modulus during loading, unloading and reloading

If the soil goes into tension, SIGMA/W arbitrarily sets E_t to 10% of the atmospheric pressure and keeps it at this value until the soil is again in compression

7 Analysis 3-Confining 100kPa

This analysis simulates consolidating the sample to an isotropic confining stress of 100 kPa. This analysis will become the “Parent” or initial condition for other subsequent tests.

The stress state is induced by applying a normal stress on the top and on the right side of the sample.

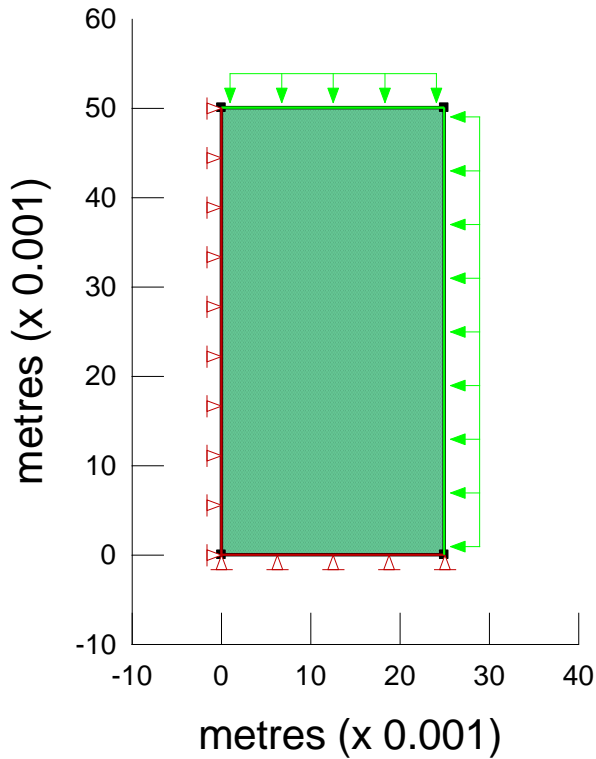


Figure 6 Inducing stress to simulate confining stress

Notice that Linear-Elastic parameters are used when setting up confining stresses; non-linear models are not required for this and the value of E is not relevant

8 Analysis A – C only with confining stress

This analysis repeats the unconfined compression test described in Section 5, except that the sample is now under a confining stress of 100 kPa.

The maximum vertical stress should now increase to 300 kPa, but the deviatoric stress ($\sigma_1 - \sigma_3$) should be the same at $(300 - 100) = 200$ kPa, making the τ_{\max} equal to 100 kPa, which is the specified shear strength. This confirmed in Figure 7 and Figure 8.

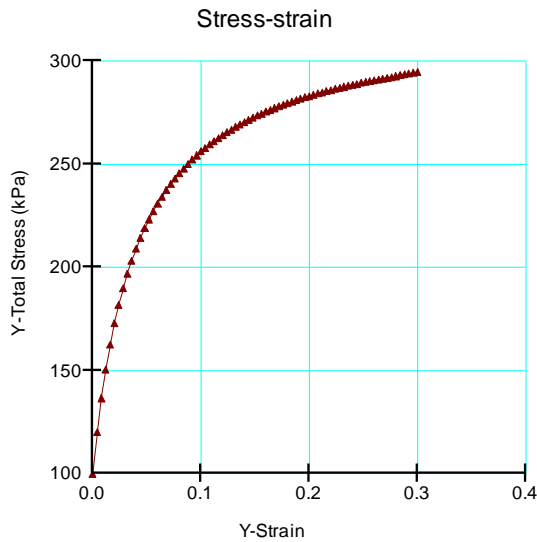


Figure 7 Vertical stress for confined test on cohesive soil

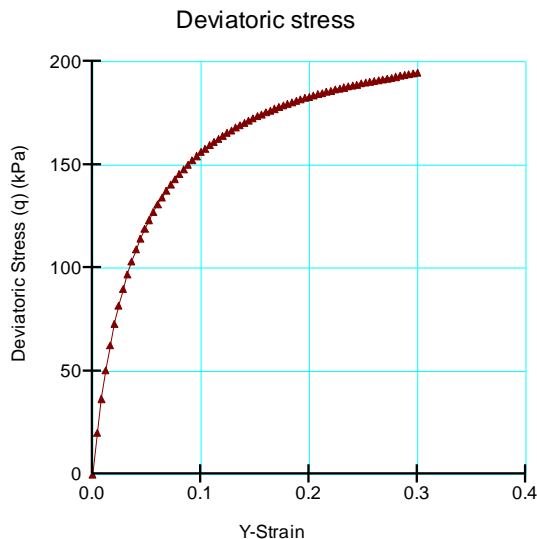


Figure 8 Deviatoric stress for confined test on cohesive soil

9 Analysis B – Frictional $\phi > 0$; $c = 0$

This analysis is a drained test on sand that has a $\phi = 30$ and $c = \text{zero}$. The confining stress again is 100 kPa.

For a purely frictional soil, the principal stress ratio at the point where the stress level reaches the soil strength is,

$$\frac{\sigma_1}{\sigma_3} = \frac{1 + \sin \phi}{1 - \sin \phi} \text{ which is equal to 3 if } \phi \text{ is 30 degrees.}$$

The confining stress (σ_3) is 100 kPa. Therefore σ_1 at failure should approach 300 kPa. Figure 9 again confirms this. Note the starting stress is 100 and migrates to 300.

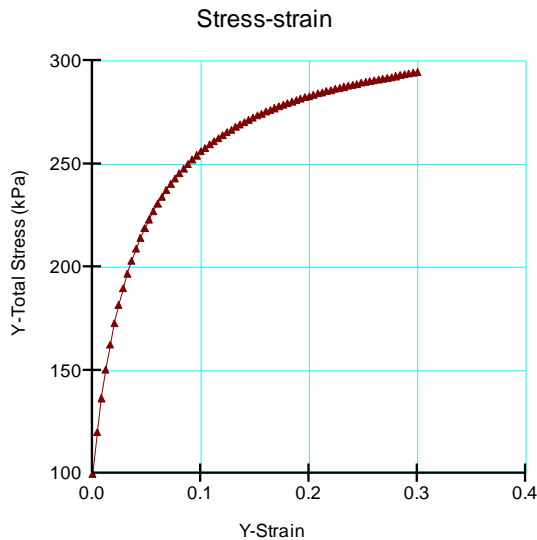


Figure 9 Vertical stress for confined test on frictional soil

10 Analysis C – Phi>0 load-reload

This analysis examines the behavior of a frictional soil under loading, unloading and then reloading. The surface loading function is the same as in Figure 3 above.

The behavior is the same as discussed earlier in Section 6 and is as intended in the SIGMA/W formulation.

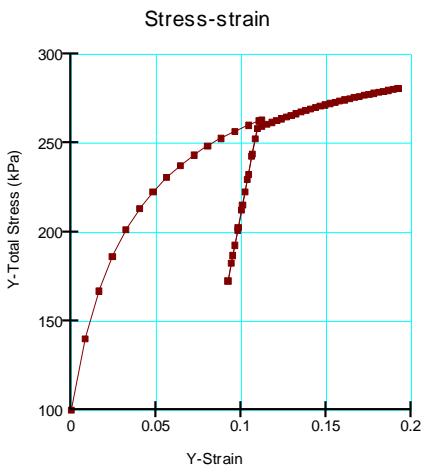


Figure 10 Loading, unloading and reloading on frictional soil

11 Analysis D – Extension

This analysis simulates an extension test.

This again is a drained test on sand that has a $\phi = 30$ and $c = \text{zero}$. The confining stress again is 100 kPa.

For a purely frictional soil, the principal stress ratio at the point where the stress level reaches the soil strength is,

$$\frac{\sigma_1}{\sigma_3} = \frac{1 + \sin \phi}{1 - \sin \phi}$$

which is equal to 3 if ϕ is 30 degrees.

The extension is simulated by pulling upward on the sample at 0.004 m per load step.

In this case, the confining stress is 100 kPa and is the major principal stress σ_1 . The vertical stress is the minor principal stress σ_3 . Therefore σ_3 at failure should approach 33.3 kPa. Figure 11 again confirms this. Note the starting stress is 100 and then drops to 33.3 as it should.

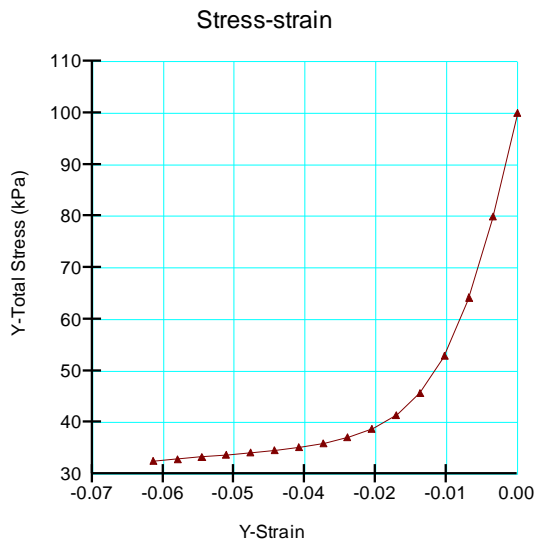


Figure 11 Unloading extension test

The deviatoric stress at failure should be $100 - 33.3 = 67.7$ kPa as is confirmed in Figure 12.

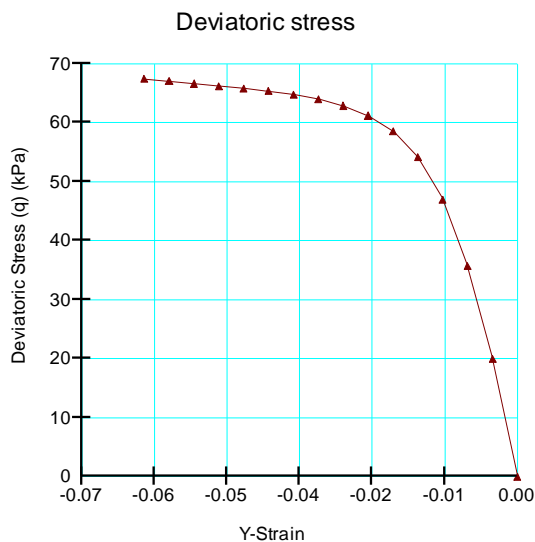


Figure 12 Deviatoric stress for extension test

12 Analysis 4 – Confining initial shear

The initial tangent modulus E_t is affected by the initial shear stress state in the soil. We can demonstrate and confirm this by simulating a triaxial test consolidated to an anisotropic stress condition. In this case the initial state of stress is induced by applying 100 kPa normal stress at the top and 50 kPa on the right side.

This makes $\sigma_1 = 100$ kPa and $\sigma_3 = 50$ kPa and the initial deviatoric stress 50 kPa.

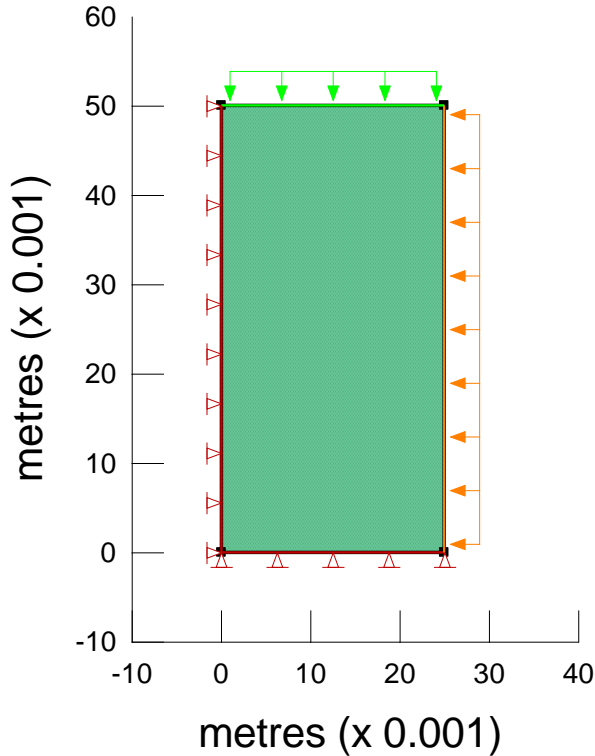


Figure 13 Configuration for anisotropic consolidation (stress state)

13 Analysis 4 – E – Frictional – initial shear

Substituting $E_i = 5000$, $R_f = 0.9$, $\phi = 30$ and the anisotropic $\sigma_1 = 100$ and $\sigma_3 = 50$ into the equation,

$$E_t = \left[1 - \frac{R_f (\sigma_1 - \sigma_3) (1 - \sin \phi)}{2c \cos \phi + 2\sigma_3 \sin \phi} \right]^2 E_i$$

makes E_t at the start of the tests equal to 1,512.50 kPa. This is

evident in Figure 14.

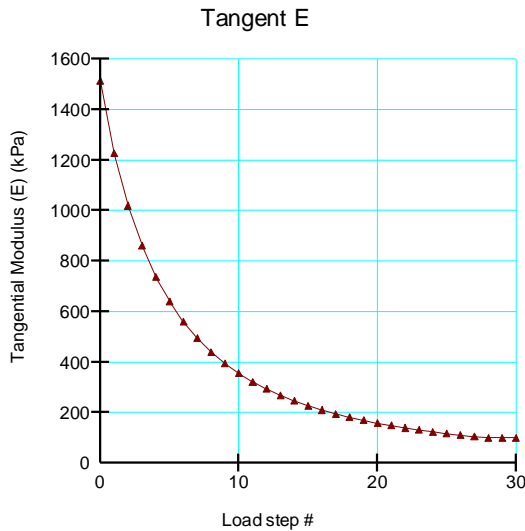


Figure 14 Starting E_t in relation to the starting shear

The fact that the starting modulus is a function of the insitu stress state is an important characteristic of the hyperbolic and needs to be kept in mind when the hyperbolic model is used.

14 Analysis 5 – Confining 100kPa /w pwp

The soil parameters for the hyperbolic model can be effective stress parameters and the Material Category then must be Effective-Drained Parameters.

The hyperbolic model can be used with effective stress parameters but the hyperbolic cannot be used in a coupled analysis where pore-pressure changes occur due to loading

In this analysis the initial consolidated effective confining stress state is established by applying a normal stress of 100 kPa on the top and on the right side of the sample. The initial back pressure of 50 kPa is specified as an Activation PWP of 50 kPa. This makes the initial effective confining pressure equal to 50 kPa.

Figure 15 shows the total and effective stress versus strain curves. The total stress goes from the initial confining stress of 100 kPa to the final vertical stress of 200 kPa. The effective stress starts at the initial effective confining stress of 50 kPa as it should, and the final effective Y-Stress approaches 150 kPa as it should, since the ratio is theoretically 3 as described earlier. The deviatoric stress at failure will be 100 kPa, which is confirmed in Figure 16.

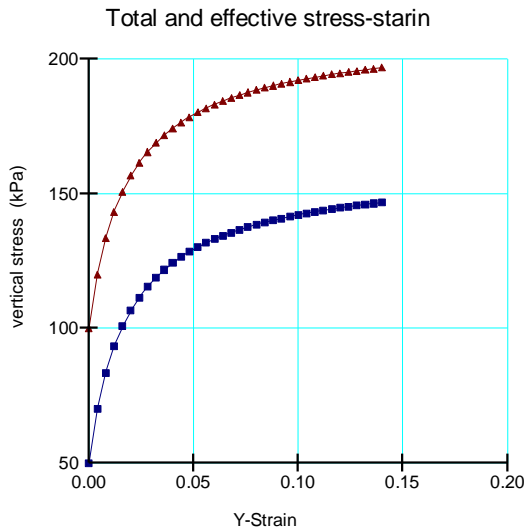


Figure 15 Total and effective stress-strain curves for test with backpressure

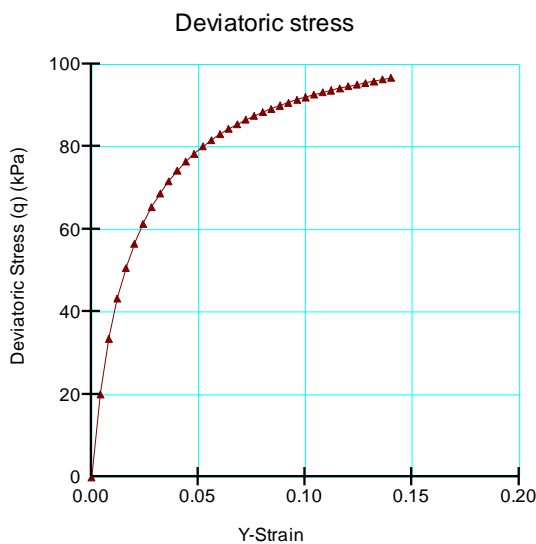


Figure 16 Deviatoric stress for test with back pressure

15 Summary

These triaxial test simulations verify that the hyperbolic constitutive relationship in SIGMA/W is functioning and behaving correctly.

The tests also highlight the following special characteristics about the SIGMA/W hyperbolic formulation:

- The minimum tangent modulus E_t allowed is equal to the atmospheric pressure
- E_i is used during unloading and re-loading until the new shear stress exceeds the previous maximum
- E_i is set to 10% of the atmospheric pressure when the soil goes into tension and is kept at this value until the soil is back in compression after some reloading

- The initial shear stress conditions in the ground affect the initial E_t used when the loading starts
- The hyperbolic model can be used with total stress or effective stress parameters, but the hyperbolic model cannot be used in a coupled analysis with pore-pressures arising from external loading