Hwang Verify Column

1 Introduction

Hwang, Murray, and Brooke, (1972), were among the first to develop a geothermal finite element model that included the effects of latent heat. This verification example problem compares Hwang's finite element solution with the TEMP/W results for a one-dimensional thaw analysis similar to the Neumann thaw and freeze problem.

2 Feature highlights

GeoStudio feature highlights include:

- Transient heat flow
- Phase change in TEMP/W
- Verification of formulation

3 Discussion

Hwang used a finite element mesh that was 5 meters high with an initial ground temperature of -2°C and an initial surface temperature of +5°C. The material properties used in the analysis are given in Table 3-1.

Table 3-1 Material properties used in Hwang analysis

Material Property	Value
Thermal conductivity	$1x10^3$ cal/m/h/°C
Specific heat capacity	1 cal/gm/°C
Unit weight (density)	$1x10^3$ kg/m^3
Volumetric heat capacity	$1x10^6$ cal/m^3/°C
Latent heat	$5x10^7$ cal/m^3
Volumetric water content	1.0 (100%)

Hwang adopted the same parameters for both the frozen and the unfrozen zones. The thaw depth at different times are tabulated in Table 3-2. In the TEMP/W model, the simplified thermal model is used. The heat units in Hwang's example are Cal, but this is not a supported unit in TEMP/W, so a generic heat unit of "H" is used in the model. The units are only for presentation purposes.

Solution

The thaw depth of the Hwang example problem can also be calculated using the graphical solution presented by Nixon and McRoberts. Since Ts = 5°C and Tg = -2°C, the (-Tg/Ts) ratio is therefore:

$$-\frac{T_g}{T_s} = \frac{-(5)}{-2} = 2.5$$

The variable Ste is calculated as:

$$Ste = \frac{C_u\ T_s}{L} = \frac{\left(1x10^6\right)\left(5\right)}{5x10^7} = 0.1$$

Using these two known variables, the normalized thaw parameter $\dfrac{\alpha}{2\sqrt{k_u}}$ can be obtained from the graph in the Freeze Column Examples.doc file as 0.2. We can then calculate α as:

$$\alpha = 2\sqrt{k_u}\ \left(0.2\right) = 2\sqrt{\left(\frac{10^3}{10^6}\right)}\left(0.2\right) = 0.0126$$

With α known, the depth of the thawing front can be computed.

The example problem is simulated in TEMP/W using 27 elements and 138 nodes. The initial temperature of the entire column is defined at -2°C using the Draw Initial Conditions command. The ground surface is specified with a constant temperature boundary (T) of 5°C. Table 3-2 compares the thaw depths calculated from the analytical solution, Hwang's finite element solution, and the TEMP/W solution for three different times. The associated files for this example are named Hwang.

Table 3-2 Comparison of calculated thaw depth from different procedures

Elapsed Time		Thaw Depth (m)		
In Hours	**In Days**	**Analytical**	**Hwang**	**TEMP/W**
126	5	0.14	0.1	0.14
1580	66	0.50	0.4	0.49
6310	263	1.00	0.9	0.92

The thaw depths computed by TEMP/W compare satisfactorily with both the analytical and Hwang solutions. Hwang's thaw depths are only available graphically, and consequently are approximated to only one-tenth of a meter.

Figure 3-1 compares the temperature profiles obtained by Hwang and by TEMP/W. Good agreement is achieved between Hwang's solution and TEMP/W solution. Except for the small differences in the temperature profiles for the last time step (6310 hours), the temperature distribution is essentially the same for both finite element formulations.

TEMP/W gives the same results as Hwang obtained using a finite element analysis. This is important because the procedure that Hwang used to formulate the latent heat component is different than the procedure used by TEMP/W. Hwang used the rate of change of frozen area to account for the latent heat, while TEMP/W uses the slope of the unfrozen water content function. The unfrozen water content function formulation is considered more appropriate, since it is more general.

The TEMP/W approach is preferred for practical field problems, since it can accommodate any gradual function; the Hwang approach can only accommodate the equivalent of a vertical stepped unfrozen water content function.

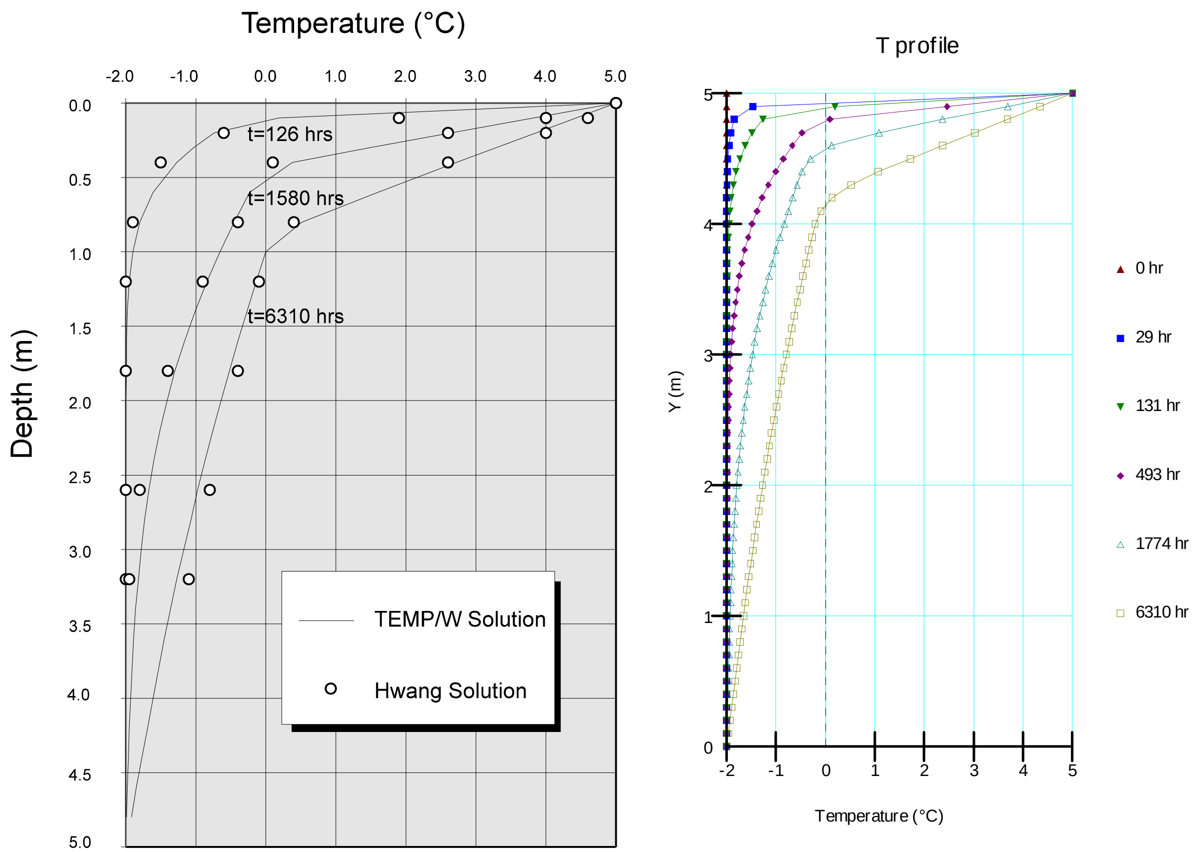

Figure 3-1 Comparison of Hwang and TEMP/W temperature profiles